NAME _____ DATE _____ CLASS _____

Holt Physics

Problem 8E

CONSERVATION OF MECHANICAL ENERGY

PROBLEM

In 1990, Eddy McDonald of Canada completed 8437 loops with a yo-yo in an hour, setting a world record. Assume that the yo-yo McDonald used has a moment of inertia of 7.50×10^{-5} kg•m^2 and a mass of 6.00×10^{-2} kg. If the yo-yo descends from a height of 6.00×10^{-1} m down a vertical string and has a linear speed of 1.80 m/s by the time it reaches the bottom of the string, what is its angular speed?

SOLUTION

1. DEFINE

Given:

$m = 6.00 \times 10^{-2}$ kg

$I = 7.50 \times 10^{-5}$ kg•m^2

$h = 6.00 \times 10^{-1}$ m

$v_f = 1.80$ m/s

$g = 9.81$ m/s^2

Unknown: $\omega_f = ?$

2. PLAN

Choose an equation(s) or situation: Apply the principle of conservation of mechanical energy.

$$ME_i = ME_f$$

Initially, the system possesses only gravitational potential energy.

$$ME_i = PE_g = mgh$$

When the yo-yo reaches the bottom of the string, this potential energy has been converted to translational and rotational kinetic energy.

$$ME_f = KE_{trans} + KE_{rot} = \tfrac{1}{2}mv_f^2 + \tfrac{1}{2}I\omega_f^2$$

$$mgh = \tfrac{1}{2}mv_f^2 + \tfrac{1}{2}I\omega_f^2$$

Rearrange the equation(s) to isolate the unknown(s):

$$\tfrac{1}{2}I\omega_f^2 = mgh - \tfrac{1}{2}mv_f^2$$

$$\omega_f = \sqrt{\frac{m(2gh - v_f^2)}{I}}$$

3. CALCULATE

Substitute the values into the equation(s) and solve:

$$\omega_f = \sqrt{\frac{(6.00 \times 10^{-2}\ \text{kg})[2(9.81\ \text{m/s}^2)(6.00 \times 10^{-1}\ \text{m}) - (1.80\ \text{m/s})^2]}{(7.50 \times 10^{-5}\ \text{kg•m}^2)}}$$

$$= \boxed{82.6\ \text{rad/s}}$$

4. EVALUATE

Because part of the yo-yo's kinetic energy is taken up by rotational kinetic energy, the translational speed of the spinning yo-yo should be smaller than the translational speed of the yo-yo in free fall without rotation.

$$v_f(\text{free fall}) = \sqrt{2gh} = 3.43\ \text{m/s} > 1.80\ \text{m/s}$$