Section 1.5 – Approximate Values of Trigonometric Functions

Objectives: 1) Given the degree measure of an angle, find decimal approximations for the six trigonometric functions
2) Given a value of a trigonometric function, find the approximate degree measure of the angle.

Exact values of trigonometric functions can be found for certain angles, such as multiples of 30°, 45°, or 60°, or for angles for which you know a point on the terminal side. But for most angles, exact values of the trigonometric functions cannot be found, even using radicals. So you will need your calculator to approximate many trigonometric functions.

Your calculator has sin, cos, and tan keys. So use your calculator to find these functions directly. If the accuracy of the answer is not specified, round off the answer to two decimal places.

Example 1: Find the sin, cos, and tan of 68.4°. Round off the answers to four significant digits.

First, make sure your calculator is in degree mode. Check your calculator by hitting the MODE button, making sure the word “degree” is highlighted.

$$\sin 68.4^\circ \approx 0.929776 \approx 0.9298 \qquad \cos 68.4^\circ \approx 0.368125 \approx 0.3681 \qquad \tan 68.4^\circ \approx 2.525712 \approx 2.526$$

Note: A “0” at the beginning of a number, such as 0.25, is not a significant digit. The “0” at the right end of 0.5340 is a significant digit. Significant digits are not the same as decimal places.

To find the csc, sec, and cot functions of an angle, you use the fact that they are the reciprocals of the other three trig functions, sin, cos, and tan.

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{y}{r}} = \frac{r}{y} \qquad \sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{x}{r}} = \frac{r}{x} \qquad \cot\theta = \frac{1}{\tan\theta} = \frac{1}{\frac{y}{x}} = \frac{x}{y}$$

Example 2: Find the csc, sec, and cot of 68.4°. Round off the answers to four decimal places.

$$\csc 68.4^\circ = \frac{1}{\sin 68.4^\circ} \approx 1.075527 \approx 1.0755$$

$$\sec 68.4^\circ = \frac{1}{\cos 68.4^\circ} \approx 2.716472 \approx 2.7165$$

$$\cot 68.4^\circ = \frac{1}{\tan 68.4^\circ} \approx 0.395928 \approx 0.3959$$

Example 3: Find the indicated function value rounded to four significant digits.

a) $\tan 23^\circ 36'$

$$23^\circ 36' = 23^\circ + 36\left(\frac{1}{60}\right)^\circ = 23^\circ + 0.6^\circ = 23.60^\circ$$

$$\tan 23^\circ 36' = \tan 23.6^\circ \approx 0.436889 \approx 0.4369$$

b) $\sec 48^\circ 13'$

$$48^\circ 13' = 48^\circ + 13\left(\frac{1}{60}\right)^\circ = 48^\circ + 0.216667^\circ \approx 48.22^\circ$$

$$\sec 48^\circ 13' \approx \sec 48.22^\circ = \frac{1}{\cos 48.22^\circ} \approx 1.500888 \approx 1.501$$

c) $\sin 67^\circ 52'$

$$67^\circ 52' = 67^\circ + 52\left(\frac{1}{60}\right)^\circ = 67^\circ + 0.866667^\circ \approx 67.87^\circ$$

$$\sin 67^\circ 52' \approx \sin 67.87^\circ \approx 0.926332 \approx 0.9263$$

Section 1.5 – Approximate Values of Trigonometric Functions (continued)

For the second objective, finding the angle measure from the function value, you must use the inverse function keys on your calculator.

The inverse function keys on your calculator are marked $\sin^{-1}$, $\cos^{-1}$, and $\tan^{-1}$. These are read as "sine inverse," "cosine inverse," and "tangent inverse."

$\sin^{-1} x$ means "An angle whose sine is x."
$\cos^{-1} x$ means "An angle whose cosine is x."
$\tan^{-1} x$ means "An angle whose tangent is x."
$\csc^{-1} x$ means "An angle whose cosecant is x."
$\sec^{-1} x$ means "An angle whose secant is x."
$\cot^{-1} x$ means "An angle whose cotangent is x."

Note that $\sin^{-1} x$ does not mean the reciprocal of $\sin x$. The exponent -1 is used to mean the function inverse, not the multiplicative inverse. So, $\sin^{-1} x \neq \frac{1}{\sin x}$. **********

Example 4: Find the measure of the acute angle θ
1) correct to 2 decimal places,
2) correct to the nearest minute.

1a)
$$\theta = \sin^{-1} 0.4362$$
$$\theta \approx 25.861676^\circ$$
$$\theta \approx 25.86^\circ$$

1b)
$$\theta = \cos^{-1} 0.7128$$
$$\theta \approx 44.536810^\circ$$
$$\theta \approx 44.54^\circ$$

1c)
$$\theta = \tan^{-1} 1.2593$$
$$\theta \approx 51.547194^\circ$$
$$\theta \approx 51.55^\circ$$

To check your answers:

$$\sin 25.86^\circ \approx 0.436174 \approx 0.4362$$
$$\cos 44.54^\circ \approx 0.712761 \approx 0.7128$$
$$\tan 51.55^\circ \approx 1.259427 \approx 1.2594$$

2a)
$$\theta \approx 25.86^\circ$$
$$\theta \approx 25^\circ + .86(60')$$
$$\theta \approx 25^\circ + 51.6'$$
$$\theta \approx 25^\circ 52'$$

2b)
$$\theta \approx 44.54^\circ$$
$$\theta \approx 44^\circ + .54(60')$$
$$\theta \approx 44^\circ + 32.4'$$
$$\theta \approx 44^\circ 32'$$

2c)
$$\theta \approx 51.55^\circ$$
$$\theta \approx 51^\circ + .55(60')$$
$$\theta \approx 51^\circ + 33'$$
$$\theta \approx 52^\circ 33'$$

To get the remaining three inverse functions, use the basic trigonometric relationships.

Since $\csc\theta = \frac{1}{\sin\theta}$, then if $\csc\theta = x$

$$\frac{1}{\sin\theta} = x$$
$$1 = x\sin\theta$$
$$\frac{1}{x} = \sin\theta$$

So, $\theta = \sin^{-1}\left(\frac{1}{x}\right)$.

Thus, to calculate the remaining three inverse functions, you need to do the following:

$$\csc^{-1} x = \sin^{-1}\left(\frac{1}{x}\right) \qquad \sec^{-1} x = \cos^{-1}\left(\frac{1}{x}\right) \qquad \cot^{-1} x = \tan^{-1}\left(\frac{1}{x}\right)$$

Section 1.5 – Approximate Values of Trigonometric Functions (continued)

Example 5: Find the measure of the acute angle θ
1) correct to 2 decimal places,
2) correct to the nearest minute.

1a) $\theta = \csc^{-1} 2.1475$
$= \sin^{-1}\left(\dfrac{1}{2.1475}\right)$
$\theta \approx 27.752798^\circ$
$\theta \approx 27.75^\circ$

1b) $\theta = \sec^{-1} 1.3892$
$= \cos^{-1}\left(\dfrac{1}{1.3892}\right)$
$\theta \approx 43.958831^\circ$
$\theta \approx 43.96^\circ$

1c) $\theta = \cot^{-1} 3.7619$
$= \tan^{-1}\left(\dfrac{1}{3.7619}\right)$
$\theta \approx 14.886285^\circ$
$\theta \approx 14.89^\circ$

2a) $\theta \approx 27.75^\circ$
$\theta \approx 27^\circ + .75(60')$
$\theta \approx 27^\circ + 45'$
$\theta \approx 27^\circ 45'$

2b) $\theta \approx 43.96^\circ$
$\theta \approx 43^\circ + .96(60')$
$\theta \approx 43^\circ + 57.6'$
$\theta \approx 43^\circ 58'$

2c) $\theta \approx 14.89^\circ$
$\theta \approx 14^\circ + .89(60')$
$\theta \approx 14^\circ + 53.4'$
$\theta \approx 14^\circ 53'$

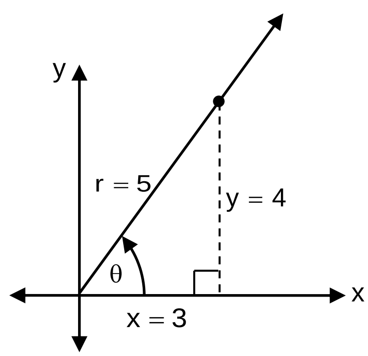
For problems 27 – 32 on page 26:

Find angle θ using your calculator and then find the given function of θ. Confirm your answer is right by sketching the angle and using the definitions of the trigonometric functions.

27)

Find $\sin\theta$ if $\cos\theta = \dfrac{3}{5}$.

$\theta = \cos^{-1}\left(\dfrac{3}{5}\right)$
$\theta \approx 53.130102^\circ$
$\theta \approx 53.13^\circ$
$\sin\theta = \sin 53.13^\circ$
≈ 0.799999
≈ 0.80



$\cos\theta = \dfrac{3}{5}$
$= \dfrac{x}{r}$

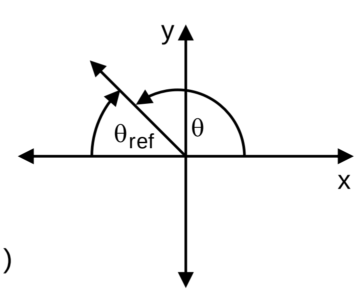
Assume: $x = 3,\ r = 5$
$x^2 + y^2 = r^2$
$y = \sqrt{r^2 - x^2}$ Quad I, y pos
$= \sqrt{5^2 - 3^2}$
$= \sqrt{25 - 9}$
$= \sqrt{16}$
$= 4$

$\sin\theta = \dfrac{y}{r}$
$= \dfrac{4}{5}$
$= 0.80$

For problems 33 – 38 on page 26:

Find the measure of θ
a) correct to 2 decimal places,
b) correct to the nearest minute.

#33) $\sin\theta = 0.6468$ and $90^\circ < \theta < 180^\circ$
$\theta_{ref} = \sin^{-1}(0.6468)$ $\quad 90^\circ < \theta < 180^\circ$
$\approx 40.300767^\circ$ $\quad \Rightarrow \theta$ in Quadrant II
$\approx 40.30^\circ$



a) $\theta = 180^\circ - \theta_{ref}$
$= 180^\circ - 40.30^\circ$
$= 139.70^\circ$

b) $\theta = 139^\circ + 0.70(60')$
$= 139^\circ + 42'$
$= 139^\circ 42'$

All material has been taken from *Trigonometry,* by P. Foerster, 3rd Edition