Section 1.6 – Approximate Values of Trigonometric Functions

Objective: Given two sides or a side and an angle of a right triangle, find measures of the other sides and angles.

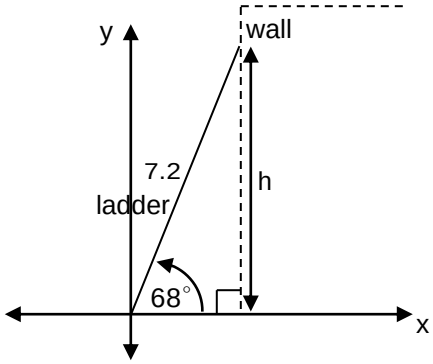
As stated before, trigonometry means triangle measurement, and it was invented for this purpose. The problems in this section are word problems that require you to create a mathematical model to represent the relationships given in the problem. These problems, each involving a right triangle, will use the six trigonometric functions that we have been working with in this chapter. A common use of trigonometry is to measure heights and distances that are either awkward or impossible to measure by ordinary means.

To solve these problems, use the following technique:

1) Draw a picture of a right triangle and identify the right angle.
2) Place the triangle with a *known* or *desired* acute angle in standard position.
3) If a distance is being sought, write a trigonometric ratio with the desired distance in the *numerator*. Then do the necessary algebra to calculate the distance.
4) If an angle is being sought, find one of its trigonometric functions by dividing the length of one known side by another. Then find the measure of the angle by using the inverse operation and your calculator.

Example 1:

You lean a ladder that is 7.2 meters long against a wall. The ladder makes an angle of 68° with the level ground. How high up is the top of the ladder?



$$\sin\theta = \frac{\text{opp}}{\text{hyp}}$$

$$\sin 68^\circ = \frac{h}{7.2}$$

$$h = 7.2\sin 68^\circ$$

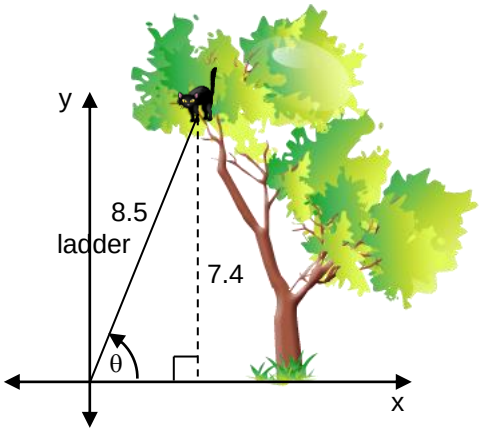
$$h \approx 6.675724 \text{ m}$$

$$h \approx 6.68 \text{ m}$$

The top of the ladder is 6.68 m above the ground.

Example 2:

Your cat is trapped on a tree branch 7.4 feet above the ground. Your ladder is 8.5 feet long. If you place the ladder's tip on the branch, what angle will the ladder make with the ground?



$$\sin\theta = \frac{\text{opp}}{\text{hyp}}$$

$$\sin\theta = \frac{7.4}{8.5}$$

$$\theta = \sin^{-1}\left(\frac{7.4}{8.5}\right)$$

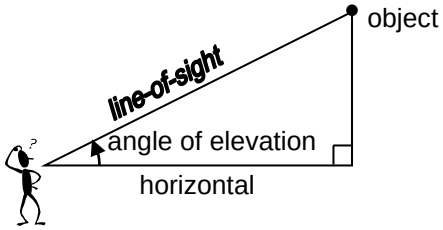
$$\theta \approx 60.527^\circ$$

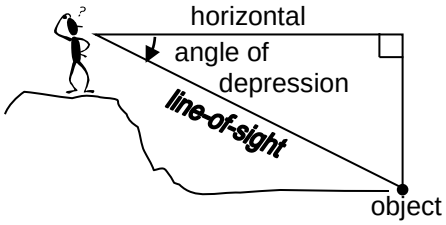
$$\theta \approx 60.53^\circ$$

The ladder will make an angle of 60.53° with the ground.

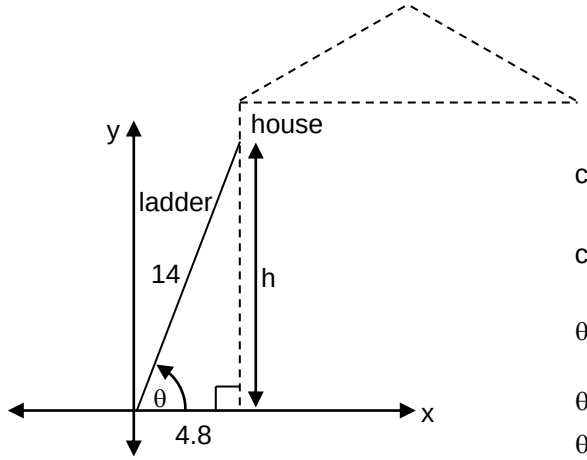
Section 1.6 – Approximate Values of Trigonometric Functions (continued)

Vertical heights can be measured using either the angle of elevation or the angle of depression. If a person is looking up at an object, the acute angle measured from the horizontal to the line-of-sight observation of the object is called the angle of elevation. If a person is looking down at an object, the acute angle made by the line-of-sight observation of the object and the horizontal is called the angle of depression.





Example 3: A ladder 14 feet long is leaning against a house. The foot of the ladder is 4.8 feet from the house. Find the angle of elevation of the ladder and the height it reaches on the house.



$$\cos\theta = \frac{\text{adj}}{\text{hyp}} \qquad \tan\theta = \frac{\text{opp}}{\text{adj}}$$

$$\cos\theta = \frac{4.8}{14} \qquad \tan 69.95^\circ = \frac{h}{4.8}$$

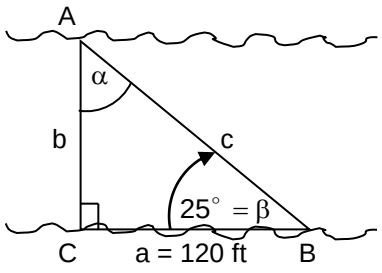
$$\theta = \cos^{-1}\left(\frac{4.8}{14}\right) \qquad h = 4.8\tan 69.95^\circ$$

$$\theta \approx 69.948^\circ \qquad h \approx 13.152169 \text{ feet}$$

$$\theta \approx 69.95^\circ \qquad h \approx 13.15 \text{ feet}$$

The angle of elevation of the ladder is 69.95°, and the ladder reaches a height of 13.15 feet on the house.

Example 4: A surveyor can measure the width of a river by setting up a transit (an instrument used in surveying to measure angles) at a point C on one side of the river and taking a sighting of a point A on the other side of the river. After turning through an angle of 90° at C, the surveyor walks a distance of 120 feet to point B. Using the transit at point B, the angle β is measured and found to be 25°. Find the width of the river.



$$\tan\beta = \text{opp}/\text{adj}$$

$$\tan 25^\circ = \frac{b}{120}$$

$$b = 120\tan 25^\circ$$

$$b = 55.956 \text{ feet}$$

$$b \approx 55.96 \text{ feet}$$

The river is approximately 55.96 feet wide.

All material has been taken from *Trigonometry,* by P. Foerster, 3rd Edition