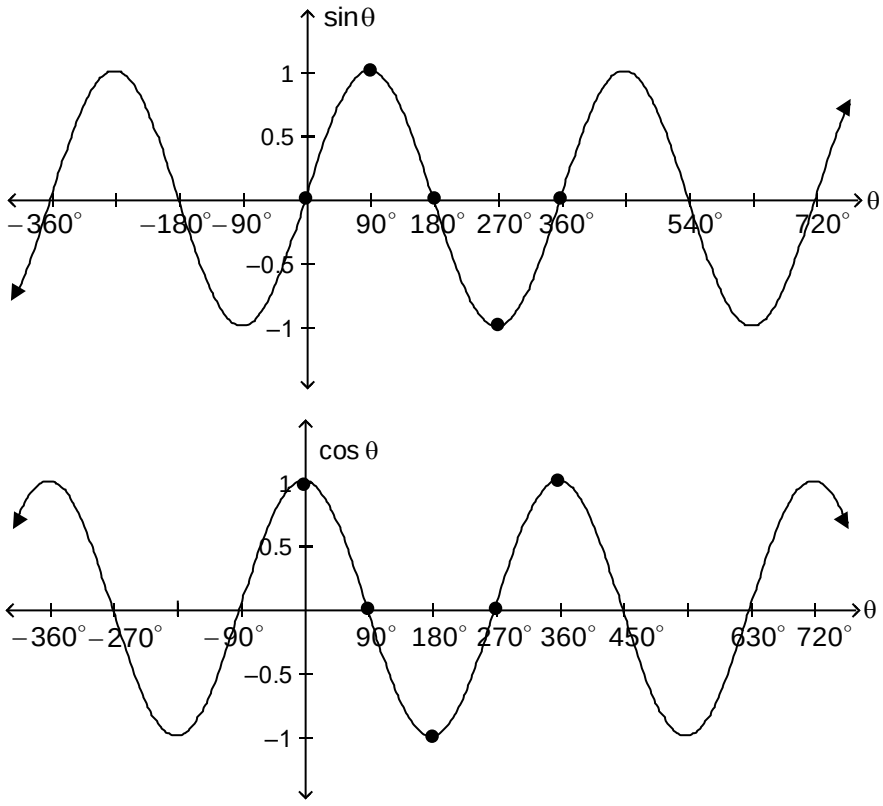
Section 2.2 – Sinusoids – Amplitude and Period

Objective: From the equation of a sinusoid, find the amplitude and the period, and use these numbers to sketch the graph.

In section 2.1, you found that the graphs of $y = \sin\theta$ and $y = \cos\theta$ look like the following:



What are the similarities between the graphs?

Differences between the graphs?

You will be drawing a lot of sine and cosine graphs, so know their basic shapes. There are five critical points for graphing each function: at θ values of 0°, 90°, 180°, 270°, and 360°, where the graph reaches a maximum, a minimum, or crosses the $\theta-$axis.

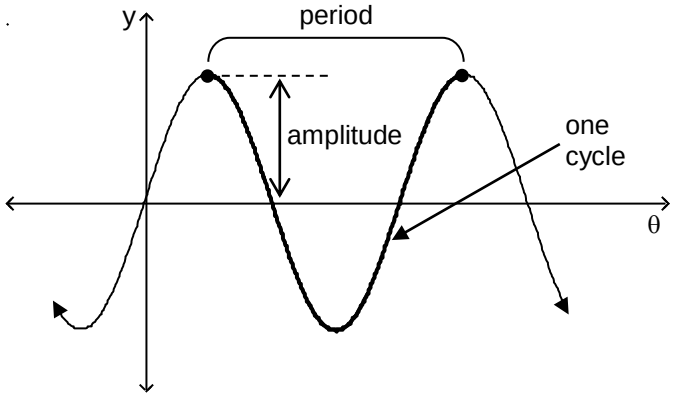
These curves are called sinusoids. These functions are called periodic functions because they repeat themselves at regular intervals, or periods. Since the sine and cosine functions repeat themselves every 360°, their period is 360°.

Parts of a sinusoid:

A cycle of a periodic function is a portion of the graph of the function from one point on the graph to the point at which the graph starts repeating itself.

The period of a trigonometric function is the number of degrees taken to complete one cycle.

The amplitude of a sinusoid is the distance from its axis to a high point or low point.



Section 2.2 – Sinusoids – Amplitude and Period (continued)

The distances to which a sinusoid rises and falls above and below the axis is its amplitude. For the sine and cosine curves above, the amplitude is 1.

A function f is periodic with period *p* if there exists some constant *p* for which $f(\theta + p) = f(p)$ for all values of θ.

You will be graphing functions of the following form:

$y = A\sin(B\theta)$ or $y = A\cos(B\theta)$, where

1) $|A|$ is the amplitude
2) B is the number of cycles the sinusoid completes in 360°.

   So, the period $P = \dfrac{360^\circ}{B}$.
3) The two constants A and B affect the graph independently.

To determine the $\theta-$values of the five critical points required to graph each sine and cosine function, take the period of your function and divide by 4. The resultant number is the $\theta-$value difference between each of the five critical points.

For example, if $B = \dfrac{1}{2}$, then the period $P = \dfrac{360^\circ}{\frac{1}{2}}$

$$P = 720^\circ$$

Calculate the step size between $\theta-$values: $\theta_{step} = \dfrac{P}{4}$

$$\theta_{step} = \frac{720^\circ}{4}$$

$$\theta_{step} = 180^\circ$$

So, the five critical $\theta-$value coordinates for graphing are 0°, $0^\circ + 180^\circ = 180^\circ$, $180^\circ + 180^\circ = 360^\circ$, $360^\circ + 180^\circ = 540^\circ$, and $540^\circ + 180^\circ = 720^\circ$.

Sketch one complete cycle of each function by finding the five critical points (high and low critical points and $\theta-$intercepts).

Example 1: $y = \cos(2\theta)$

$y = A\cos(B\theta) \Rightarrow A = 1,\ B = 2$

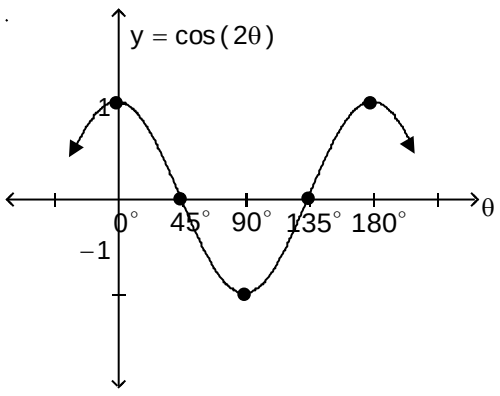
Amplitude $Amp = |A| = |1| = 1$

Period $P = \dfrac{360^\circ}{B} = \dfrac{360^\circ}{2} = 180^\circ$

$\theta_{step} = \dfrac{P}{4} = \dfrac{180^\circ}{4} = 45^\circ$

Critical Points

θ	$y = \cos(2\theta)$
0°	1
45°	0
90°	-1
135°	0
180°	1



Section 2.2 – Sinusoids – Amplitude and Period (continued)

Example 2: $y = 3\cos(2\theta)$

$y = A\cos(B\theta) \Rightarrow A = 3,\ B = 2$

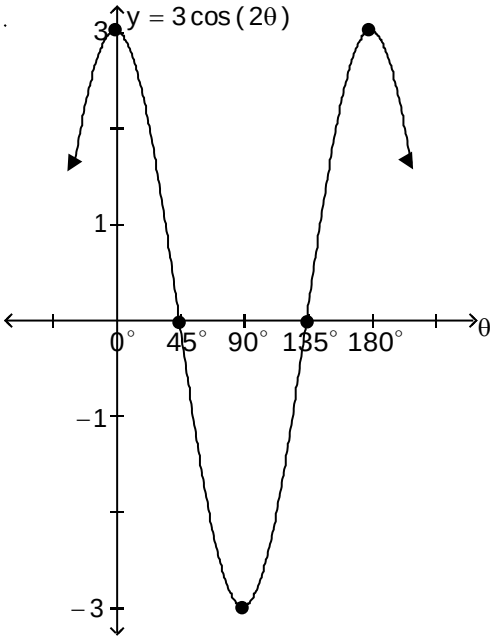
Amplitude $Amp = |A|$ Period $P = \frac{360^\circ}{B}$ $\theta_{step} = \frac{P}{4}$

$= |3|$ $= \frac{360^\circ}{2}$ $= \frac{180^\circ}{4}$

$= 3$ $= 180^\circ$ $= 45^\circ$

Critical Points

θ	$y = 3\cos(2\theta)$
0°	3
45°	0
90°	-3
135°	0
180°	3



Example 3: $y = 2\sin\left(\frac{1}{3}\theta\right)$

$y = A\sin(B\theta) \Rightarrow A = 2,\ B = \frac{1}{3}$

Amplitude $Amp = |A|$ Period $P = \frac{360^\circ}{B}$ $\theta_{step} = \frac{P}{4}$

$= |2|$ $= \frac{360^\circ}{\frac{1}{3}}$ $= \frac{1080^\circ}{4}$

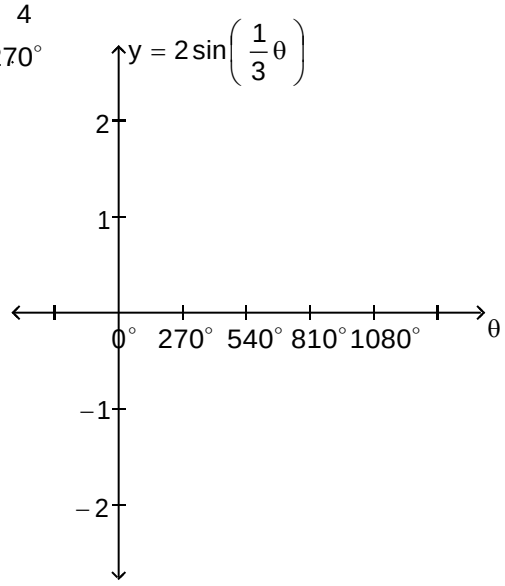
$= 2$ $= 3(360^\circ)$ $= 270^\circ$

$= 1080^\circ$

Draw the graph:

Critical Points

θ	$y = 2\sin\left(\frac{1}{3}\theta\right)$
0°	0
270°	2
540°	0
810°	-2
1080°	0



Using $y = A\sin(B\theta)$ or $y = A\cos(B\theta)$:

From the examples above, you can see that when the amplitude $|A|$ is not 1, but say $|A| = 2$, then the function has been vertically stretched (magnified or dilated) by a factor of 2. In this case, all the basic function's y-values have been multiplied by 2.

When B is not 1, then the function gets horizontally stretched or compressed. In this case, all the basic function's x-values have changed. If B is greater than 1, then the period of the function becomes smaller than 360°, so the function repeats more often. If B is between 0 and 1, then the period of the function becomes greater than 360°, so the function repeats less often. Remember that B represents the number of cycles the function completes in 360°.

All material has been taken from *Trigonometry,* by P. Foerster, 3rd Edition