Section 2.3 – Sinusoids – Phase Shift and Vertical Shift

Objective: Be able to draw graphs of functions such as $y = \cos(\theta + 60^\circ)$ and $y = 3 + \cos\theta$.

In section 2.2 you learned that the constants A and B in $y = A\sin(B\theta)$ and $y = A\cos(B\theta)$ are multiplied by the function value or argument. Constant A caused the graph of the equation to be vertically stretched by a factor of A. Constant B caused the graph to be horizontally stretched (for $0 < B < 1$) or compressed (for $B > 1$).

Remember, if $y = A\sin(B\theta)$ and $y = A\cos(B\theta)$,

$$|A| = \text{Amplitude}, \quad \text{Period} = \frac{360^\circ}{B}, \text{ and } \theta_{\text{step}} = \frac{\text{Period}}{4}.$$

In this section you look at what happens to the graphs if constants are added rather than multiplied.

In this section, Section 2.3, the equations are $y = C + \sin(\theta - D)$ and $y = C + \cos(\theta - D)$ where

1) C is the vertical displacement (moving up or down).
2) D is the phase displacement (moving right or left).
3) The two constants C and D affect the graph independently.

We will use the following notation (a little bit different than the text notation):
the equations are $y = VD + \sin(\theta - PD)$ and $y = VD + \cos(\theta - PD)$ where

1) VD is the vertical displacement (moving up or down).
2) PD is the phase displacement (moving right or left).
3) The two constants VD and PD affect the graph independently.

The phase displacement (PD) of a sinusoid is the value of θ that makes the argument of the sine or cosine equal to zero.

The axis running through the "middle" of the graph is called the sinusoidal axis

Example 1: Sketch one complete cycle of the graph by finding high and low critical points and points where the graph crosses the sinusoidal axis.

$$y = \cos(\theta + 30^\circ)$$
$$y = VD + \cos(\theta - PD)$$
$$\Rightarrow \; VD = 0 \text{ and } PD = -30^\circ$$

You want to find values of θ that make the cosine function's argument equal to $0^\circ, 90^\circ, 180^\circ$, 270°, and 360°.

To determine the table's Θ-values:

$\theta + 30^\circ = 0^\circ$
$\theta = -30^\circ$

$\theta + 30^\circ = 90^\circ$
$\theta = 60^\circ$

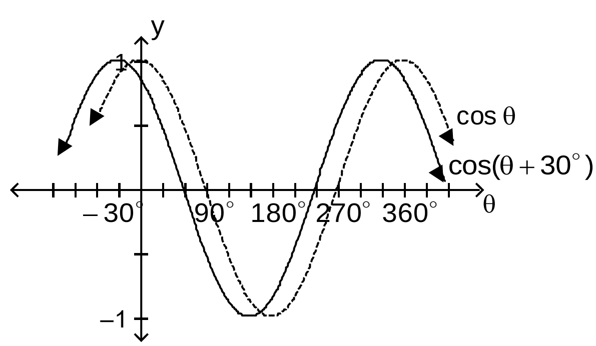
$\theta + 30^\circ = 180^\circ$
$\theta = 150^\circ$

$\theta + 30^\circ = 270^\circ$
$\theta = 240^\circ$

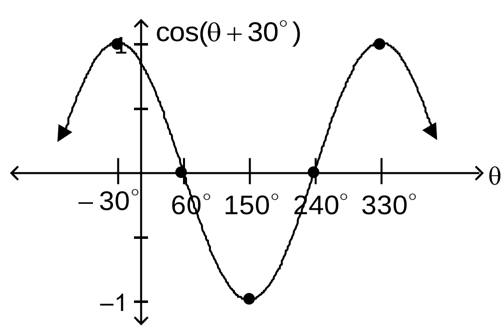
$\theta + 30^\circ = 360^\circ$
$\theta = 330^\circ$

Critical Points

θ	$\theta + 30^\circ$	$\cos(\theta + 30^\circ)$
-30°	0°	1
60°	90°	0
150°	180°	−1
240°	270°	0
330°	360°	1



The phase displacement $PD = -30^\circ$, so the cosine curve has been shifted 30° to the left.



Section 2.3 – Sinusoids – Phase Shift and Vertical Shift (continued)

Example 2: Sketch one complete cycle of the graph by finding high and low critical points and points where the graph crosses the sinusoidal axis.

$y = 2 + \sin\theta$

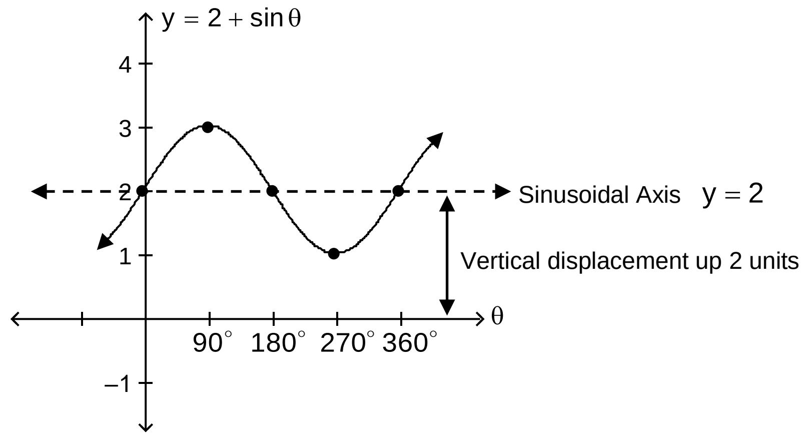
$y = VD + \sin(\theta - PD)$

$\Rightarrow\ VD = 2$ and $PD = 0^\circ$

The $VD = 2$ constant is added after the sine is found. The VD's effect is to increase each y-value of the basic sine graph by 2 units, so there is a vertical displacement up 2 units.

Critical Points

θ	$y = 2 + \sin\theta$
0°	2
90°	3
180°	2
270°	1
360°	2



In this example, the sinusoidal axis shifts up 2 units from its original (basic) position on the $\theta-$axis. The shape of the graph does not change.

Example 3: Sketch one complete cycle of the graph by finding high and low critical points and points where the graph crosses the sinusoidal axis.

$y = 1 + \sin(\theta - 40^\circ)$

$y = VD + \sin(\theta - PD)$

$\Rightarrow\ VD = 1$ and $PD = 40^\circ$

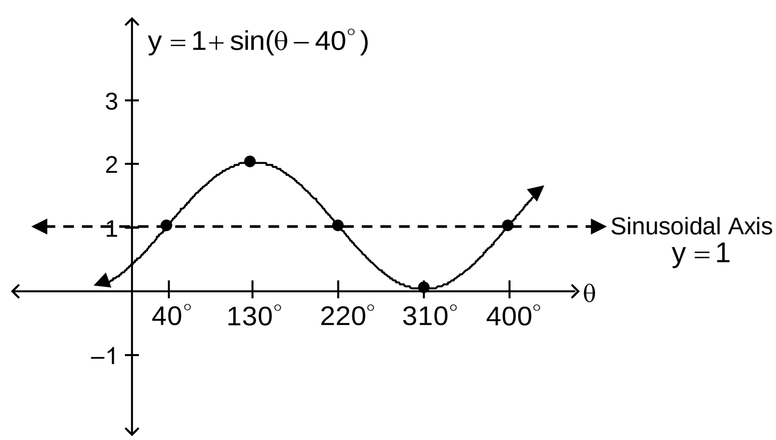
The vertical displacement $VD = 1,$ so the sine graph has been shifted up 1 unit.

The phase displacement $PD = 40^\circ,$ so the sine graph has been shifted 40° to the right.

You want to find values of θ that make the sine function's argument equal to 0°, 90°, 180°, 270°, and 360°.

Critical Points

θ	$\theta - 40^\circ$	$y = 1 + \sin(\theta - 40^\circ)$
40°	0°	1
130°	90°	2
220°	180°	1
310°	270°	0
400°	360°	1



To determine the table's θ-values:

$\theta - 40^\circ = 0^\circ$ $\quad$ $\theta - 40^\circ = 90^\circ$ $\quad$ $\theta - 40^\circ = 180^\circ$

$\theta = 40^\circ$ $\quad$ $\theta = 130^\circ$ $\quad$ $\theta = 220^\circ$

$\theta - 40^\circ = 270^\circ$ $\quad$ $\theta - 40^\circ = 360^\circ$

$\theta = 310^\circ$ $\quad$ $\theta = 400^\circ$

Section 2.3 – Sinusoids – Phase Shift and Vertical Shift (continued)

Example 4: Sketch one complete cycle of the graph by finding high and low critical points and points where the graph crosses the sinusoidal axis.

$$y = -2 + \cos(\theta + 60^\circ)$$
$$y = VD + \cos(\theta - PD)$$
$$\Rightarrow \; VD = -2 \text{ and } PD = -60^\circ$$

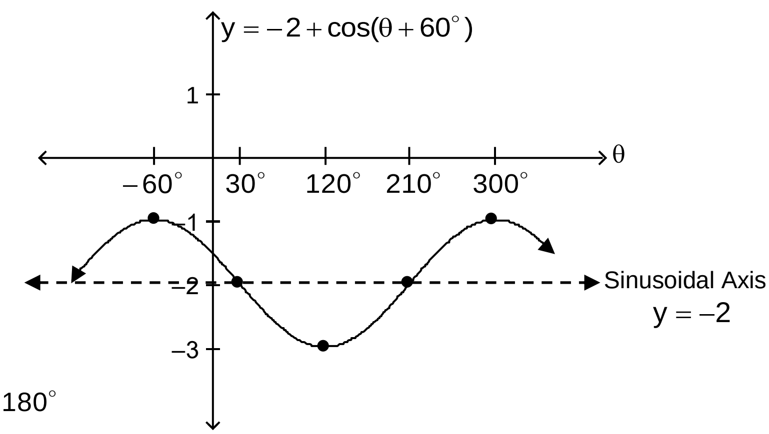
The vertical displacement $VD = -2$, so the cosine graph has been shifted down 2 units.

The phase displacement $PD = -60^\circ$, so the cosine graph has been shifted 60° to the left.

You want to find values of θ that make the cosine function's argument equal to 0°, 90°, 180°, 270°, and 360°.

Critical Points

θ	$\theta + 60^\circ$	$y = -2 + \cos(\theta + 60^\circ)$
-60°	0°	-1
30°	90°	-2
120°	180°	-3
210°	270°	-2
300°	360°	-1



To determine the table's θ-values:

$\theta + 60^\circ = 0^\circ$ $\quad$ $\theta + 60^\circ = 90^\circ$ $\quad$ $\theta + 60^\circ = 180^\circ$

$\theta = -60^\circ$ $\quad$ $\theta = 30^\circ$ $\quad$ $\theta = 120^\circ$

$\theta + 60^\circ = 270^\circ$ $\quad$ $\theta + 60^\circ = 360^\circ$

$\theta = 210^\circ$ $\quad$ $\theta = 300^\circ$