Section 2.4 – General Sinusoidal Graphs

Objective: Given any one of the following sets of information about a sinusoid, find the other two:

1) the equation
2) the graph
3) the amplitude, period or frequency, phase displacement, and vertical displacement

In this section, you will put together the ideas from Sections 2.2 and 2.3 to graph functions whose equations have the form $y = VD + A\sin\left[B(\theta - PD)\right]$ or $y = VD + A\cos\left[B(\theta - PD)\right]$.

The four constants A, B, VD, and PD have the following effects:

1) $|A|$ is the amplitude. The absolute value is needed since the constant A may be a negative number.
   So, $\text{Amplitude} = |A|$.
2) B is the number of cycles the sinusoid makes in 360°, so the period $P = \dfrac{360^\circ}{B}$.
3) VD is the vertical displacement.
4) PD is the phase displacement.

If B is negative, use the absolute value of B in the period calculation. Your text's problems do not have negative B values, but you may run into negative B values in the future.

The period is the number of degrees per cycle. It is sometimes convenient to speak of the number of cycles per degree. This quantity is called the frequency.

The frequency of a periodic function is the reciprocal of the period. So, $\text{frequency} = \dfrac{1}{\text{period}}$ and $\text{period} = \dfrac{1}{\text{frequency}}$.

An efficient stepwise procedure for drawing a sinusoid is:

1) Draw the sinusoidal axis.
2) Draw upper and lower bounds by going $|A|$ units above and below the sinusoidal axis.
3) Find the starting point of a cycle at $\theta = PD$, the phase displacement.
   Cosine functions start a cycle at a high point. Sine functions start a cycle on the sinusoidal axis, heading up.
4) The cycle will end one period later at $\theta = PD + \text{Period}$.
5) Halfway between two high points will be a low point. Halfway between each high and low point, the graph will cross the sinusoidal axis.
6) After graphing the five critical points, sketch the graph through these five critical points.

Example 1: Find the period, amplitude, frequency, phase displacement, and vertical displacement. Then use this information to find critical points and sketch the graph.

Case 1: Given an equation

$y = 5 + 3\cos\left[2(\theta - 20^\circ)\right]$

$y = VD + A\cos\left[B(\theta - PD)\right]$

$\Rightarrow A = 3,\ B = 2,\ VD = 5,$ and $PD = 20^\circ$

$\text{Amplitude} = |A| = |3| = 3$

$\text{Period } P = \dfrac{360^\circ}{B} = \dfrac{360^\circ}{2} = 180^\circ$

$\text{Frequency} = \dfrac{1}{P} = \dfrac{1}{180}$ cycle per degree

$\theta_{step} = \dfrac{P}{4} = \dfrac{180^\circ}{4} = 45^\circ$

Phase displacement: $PD = 20^\circ \Rightarrow$ shift right 20°

Vertical displacement: $VD = 5 \Rightarrow$ shift up 5 units, so the sinusoidal axis is at $y = 5$

Section 2.4 – General Sinusoidal Graphs (continued)

Example 1 continued:

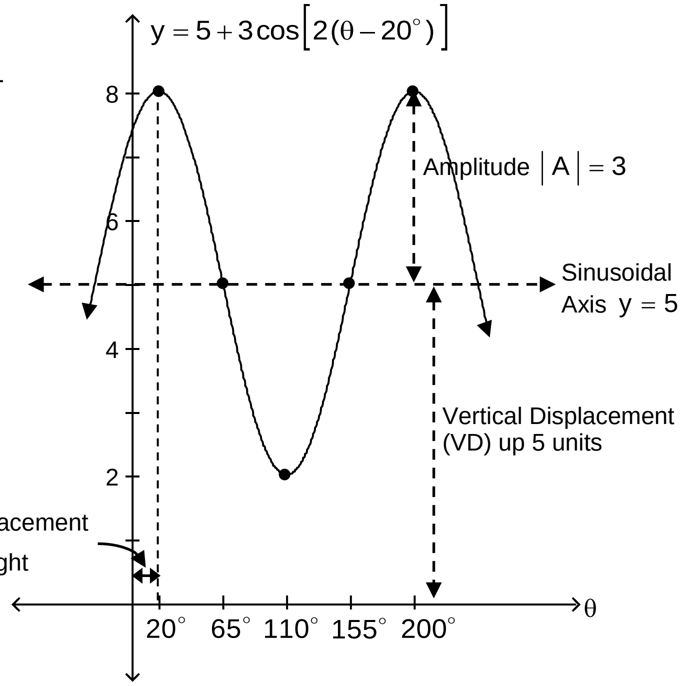
You want to find values of θ that make the argument equal to 0°, 90°, 180°, 270°, and 360°.

Set					
	$2(\theta-20^\circ)=0^\circ$	$2(\theta-20^\circ)=90^\circ$	$2(\theta-20^\circ)=180^\circ$	$2(\theta-20^\circ)=270^\circ$	$2(\theta-20^\circ)=360^\circ$
	$\theta-20^\circ=0^\circ$	$\theta-20^\circ=45^\circ$	$\theta-20^\circ=90^\circ$	$\theta-20^\circ=135^\circ$	$\theta-20^\circ=180^\circ$
	$\theta=20^\circ$	$\theta=65^\circ$	$\theta=110^\circ$	$\theta=155^\circ$	$\theta=200^\circ$

Critical Points

θ	$2(\theta-20^\circ)$	$y=5+3\cos\left[2(\theta-20^\circ)\right]$
20°	0°	8
65°	90°	5
110°	180°	2
155°	270°	5
200°	360°	8

Remember, $\theta_{step}=45^\circ$, so the θ-values are 45° apart.



Example 2: Find the period, amplitude, frequency, phase displacement, and vertical displacement. Then use this information to find critical points and sketch the graph.

Case 1: Given an equation

$$y=-3+2\sin\left[\frac{1}{2}(\theta+30^\circ)\right]$$

$$y=VD+A\sin\left[B(\theta-PD)\right]$$

$$\Rightarrow\ A=2,\ B=\frac{1}{2},\ VD=-3,\ \text{and}\ PD=-30^\circ$$

Amplitude $=|A| = |2| = 2$

Period $P=\dfrac{360^\circ}{B}=\dfrac{360^\circ}{\frac{1}{2}}=2(360^\circ)=720^\circ$

Frequency $=\dfrac{1}{P}=\dfrac{1}{720}$ cycle per degree

$\theta_{step}=\dfrac{P}{4}=\dfrac{720^\circ}{4}=180^\circ$

Phase displacement: $PD=-30^\circ\ \Rightarrow$ shift left 30°

Vertical displacement: $VD=-3\ \Rightarrow$ shift down 3 units, so the sinusoidal axis is at $y=-3$

Section 2.4 – General Sinusoidal Graphs (continued)

Example 2 continued:

You want to find values of θ that make the argument equal to 0°, 90°, 180°, 270°, and 360°.

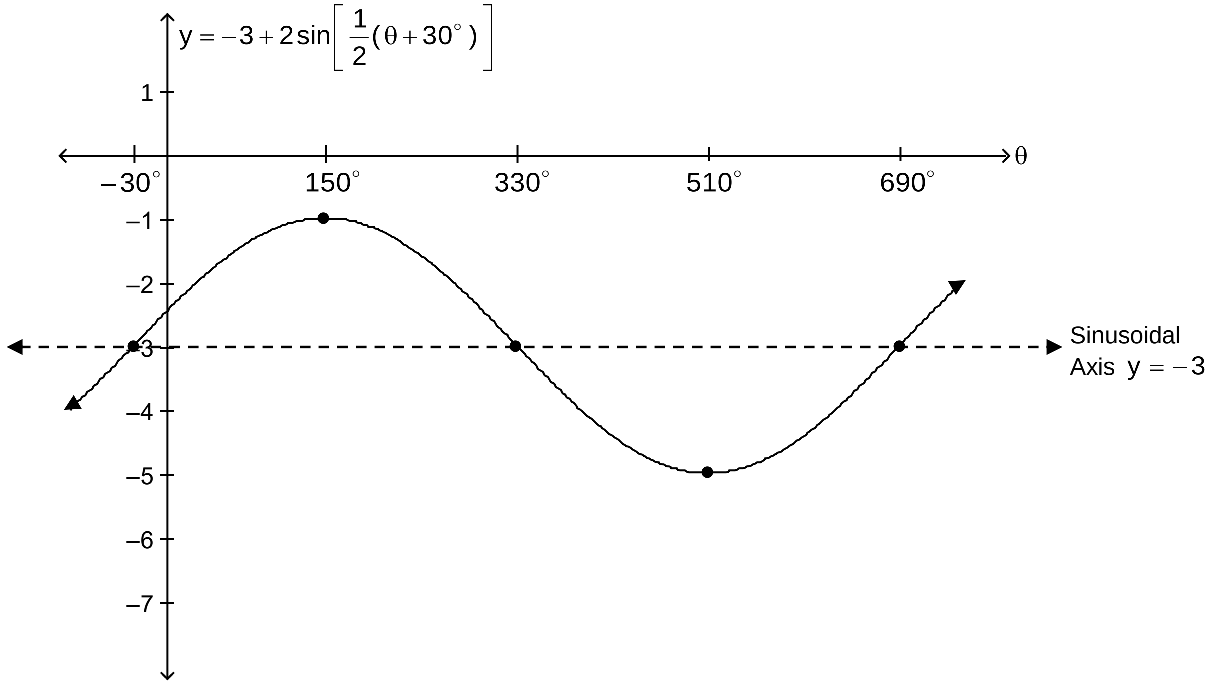
Set

$\frac{1}{2}(\theta+30^\circ)=0^\circ$	$\frac{1}{2}(\theta+30^\circ)=90^\circ$	$\frac{1}{2}(\theta+30^\circ)=180^\circ$	$\frac{1}{2}(\theta+30^\circ)=270^\circ$	$\frac{1}{2}(\theta+30^\circ)=360^\circ$
$\theta+30^\circ=0^\circ$	$\theta+30^\circ=180^\circ$	$\theta+30^\circ=360^\circ$	$\theta+30^\circ=540^\circ$	$\theta+30^\circ=720^\circ$
$\theta=-30^\circ$	$\theta=150^\circ$	$\theta=330^\circ$	$\theta=510^\circ$	$\theta=690^\circ$

Critical Points

θ	$\frac{1}{2}(\theta+30^\circ)$	$y=-3+2\sin\left[\frac{1}{2}(\theta+30^\circ)\right]$
-30°	0°	–3
150°	90°	–1
330°	180°	–3
510°	270°	–5
690°	360°	–3

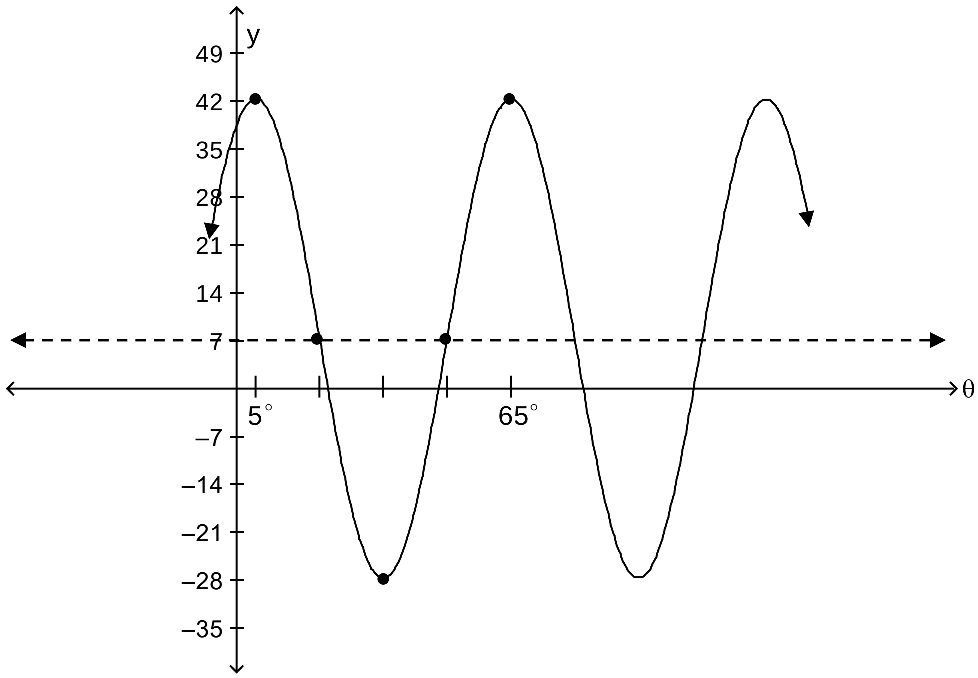
Remember, $\theta_{step}=180^\circ$, so the θ-values are 180° apart.



Section 2.4 – General Sinusoidal Graphs (continued)

Example 3: For the sinusoid sketched, determine the period, frequency, amplitude, phase displacement, and vertical displacement. Then write an equation for the sinusoid.

Case 2: Given a graph



Assume the graph is of a cosine function, since a cosine graph starts a cycle at a high point.

$y = VD + A\cos\left[B(\theta - PD)\right]$

One complete cycle begins at 5° and ends at 65°. So, the period is $65^\circ - 5^\circ = 60^\circ$.

The frequency is $\text{Frequency} = \dfrac{1}{\text{Period}}$

$= \dfrac{1}{60}$ cycle per degree

The sinusoidal axis is halfway between the upper bound (UB), 42, and the lower bound (LB), -28.
So, the vertical displacement is the average of 42 and -28.

$$\therefore VD = \frac{UB + LB}{2} = \frac{42 + -28}{2} = \frac{14}{2} = 7 \text{ units}$$

The sinusoidal axis is $y = 7$.

The amplitude is the distance between the sinusoidal axis and the upper bound and it is a non-reflected cosine graph (thus, A is positive), $\therefore A = 42 - 7$

$= 35$ units

Assuming the graph to be of a cosine function, the phase displacement is 5°, so $PD = 5$.

Since the period is 60°, $B = 360^\circ / \text{Period}$

$$= \frac{360^\circ}{60^\circ} = 6$$

So, an equation of the sinusoid graphed is $y = VD + A\cos\left[B(\theta - PD)\right]$

$$y = 7 + 35\cos\left[6(\theta - 5^\circ)\right]$$

Section 2.4 – General Sinusoidal Graphs (continued)

Example 4: Draw a graph and find an equation of the sinusoid described.

Case 3: Given the Amplitude, Period, PD, and VD

Period $= 90^\circ$, amplitude $= 2$ units, phase displacement (for a sine function) equals 30°, vertical displacement $= 3$ units.

$$B = \frac{360^\circ}{\text{Period}} = \frac{360^\circ}{90^\circ}$$
$$B = 4$$

Assuming a non-reflected sine function, so $A = 2$.

$PD = 30^\circ \quad VD = 3$

So, the equation is

$$y = VD + A\sin\left[B(\theta - PD)\right]$$
$$y = 3 + 2\sin\left[4(\theta - 30^\circ)\right]$$

You want to find values of θ that make the argument equal to 0°, 90°, 180°, 270°, and 360°.

Set $4(\theta - 30^\circ) = 0^\circ$ | $4(\theta - 30^\circ) = 90^\circ$ | $4(\theta - 30^\circ) = 180^\circ$ | $4(\theta - 30^\circ) = 270^\circ$ | $4(\theta - 30^\circ) = 360^\circ$

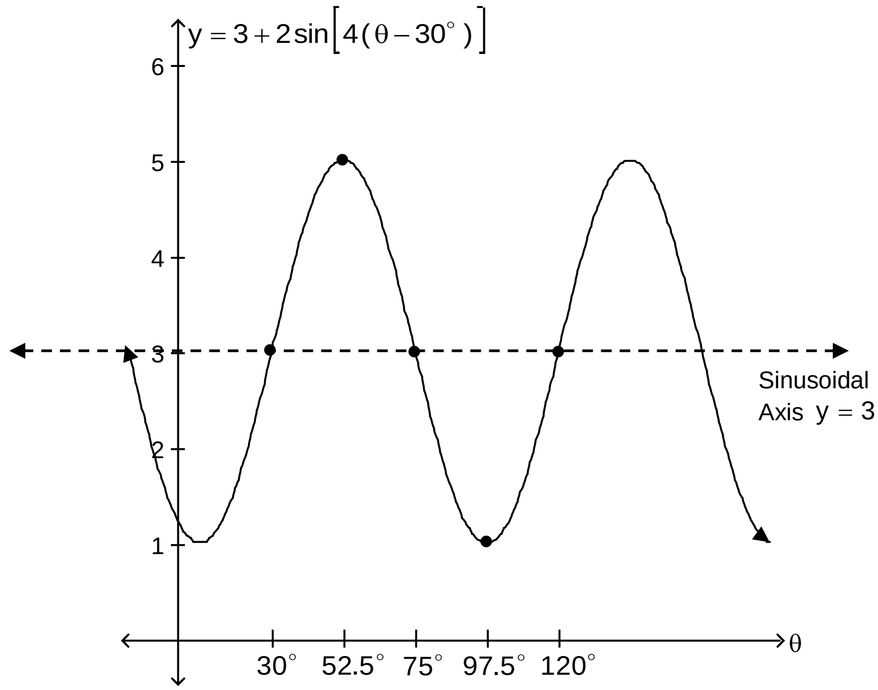
$\theta - 30^\circ = 0^\circ$ | $\theta - 30^\circ = 22.5^\circ$ | $\theta - 30^\circ = 45^\circ$ | $\theta - 30^\circ = 67.5^\circ$ | $\theta - 30^\circ = 90^\circ$

$\theta = 30^\circ$ | $\theta = 52.5^\circ$ | $\theta = 75^\circ$ | $\theta = 97.5^\circ$ | $\theta = 120^\circ$

$$\theta_{step} = \frac{P}{4} = \frac{90^\circ}{4} = 22.5^\circ$$

Critical Points

θ	$4(\theta - 30^\circ)$	$y = 3 + 2\sin\left[4(\theta - 30^\circ)\right]$
30°	0°	3
52.5°	90°	5
75°	180°	3
97.5°	270°	1
120°	360°	3



All material has been taken from *Trigonometry,* by P. Foerster, 3rd Edition