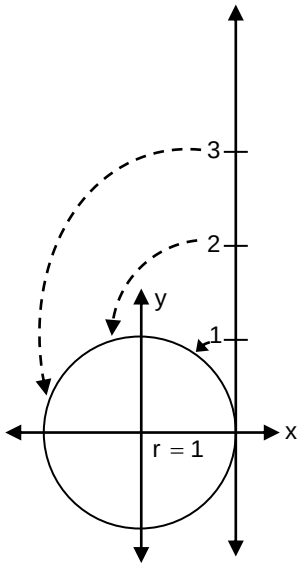
Section 2.5 – Radian Measure of Angles

Objectives: 1) Given the measure of an angle in degrees, find its measure in radians, and vice versa.
2) Find trigonometric function values for angles in radians.

The Babylonians were probably the first to divide a circle into 360 equal parts, called degrees, because they had a base-60 numbering system. A circle can be divided into any number of parts. Besides degrees, another common way to divide a circle is by using radians.

A radian is a unit of angular measurement derived by wrapping a number line around a unit circle (a circle with a radius of 1 unit).



Since the circumference of a circle is $2\pi r$, and r for a unit circle is 1, the wrapped number line divides the circle into 2π, or a bit more than 6, parts. A central angle that cuts off one unit of arc length has a measure of one radian. The radian measure of any angle is equal to the arc length cut off on a unit circle centered at the vertex of the angle.

Since we will be using degrees and radians as units of angular measurement, it is important to distinguish between the two types of units when writing an angular measurement. When an angle is measured in degrees, the degree symbol will be shown. When an angle is measured in radians, the word radians is usually omitted. So, the measure of an angle without units is understood to mean radians.

Your textbook uses the following notation: $m^{\circ}(\theta) =$ degree measure of angle θ
$m^{R}(\theta) =$ radian measure of angle θ

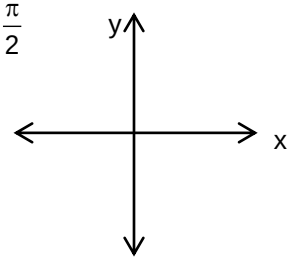
One complete revolution is said to measure 360° or 2π radians.

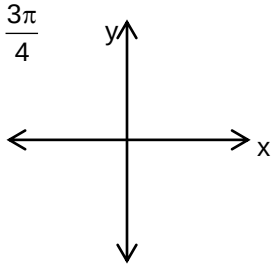
$\Rightarrow \quad 360^{\circ} = 2\pi$ radians

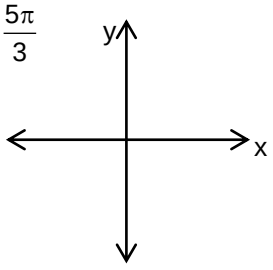
$\Rightarrow \quad 180^{\circ} = \pi$ radians

$\Rightarrow \quad 90^{\circ} = \frac{\pi}{2}$ radians

Example 1: Draw angles of the following measures:

a) $\frac{\pi}{2}$ 

b) $\frac{3\pi}{4}$ 

c) $\frac{5\pi}{3}$ 

Section 2.5 – Radian Measure of Angles (continued)

We have the following conversion factors to convert from degrees to radians and radians to degrees.

1 degree = $\frac{\pi}{180}$ radians 1 radian = $\frac{180}{\pi}$ degrees

Example 2: Convert each angle in degrees to radians.

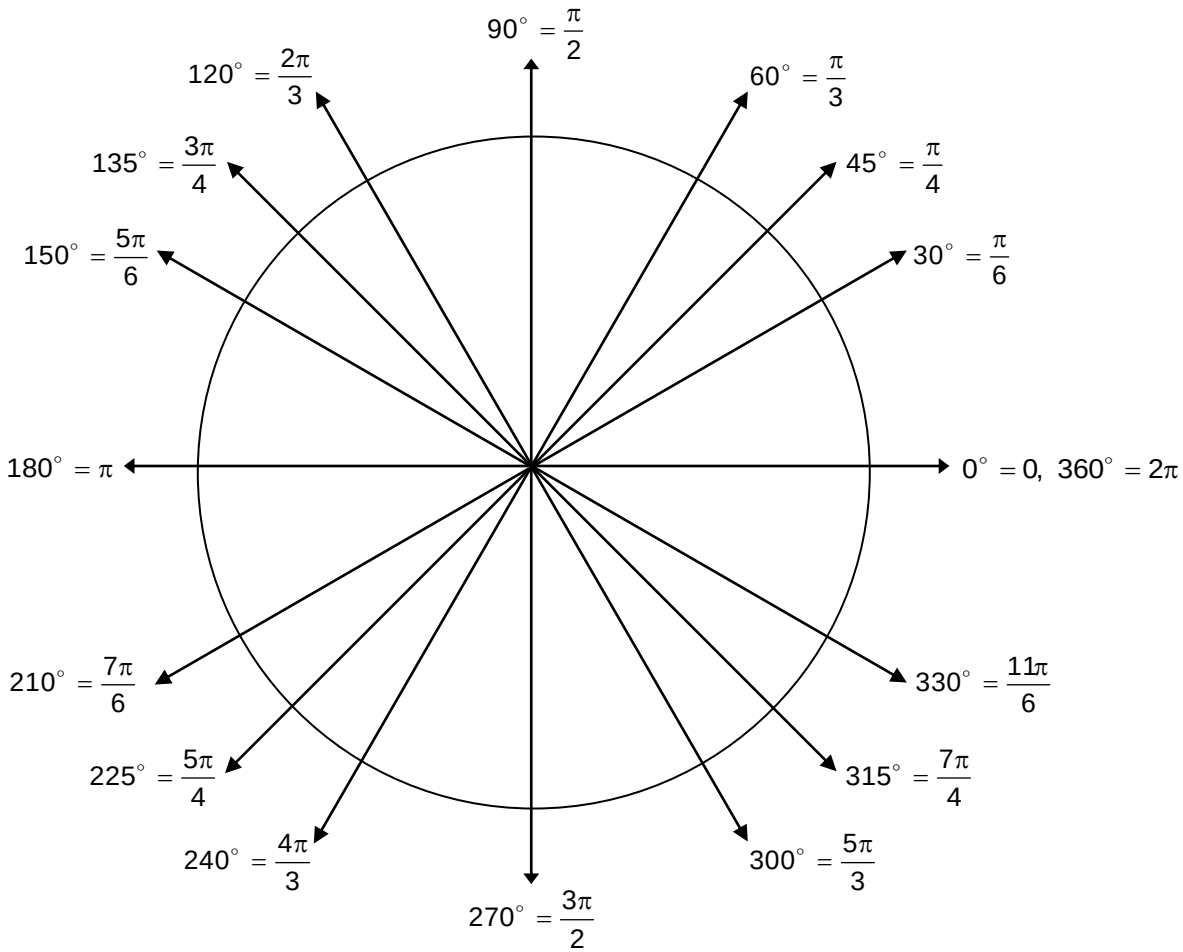
a) $80^\circ = 80(1 \text{ degree})$
$= 80\left(\frac{\pi}{180}\right)$ radians
$= \frac{4}{9}\pi$ radians

b) $-120^\circ = -120(1 \text{ degree})$
$= -120\left(\frac{\pi}{180}\right)$ radians
$= \frac{-2}{3}\pi$ radians

Example 3: Convert each angle in radians to degrees.

a) $\frac{\pi}{4}$ radians $= \frac{\pi}{4}(1 \text{ radian})$
$= \frac{\pi}{4}\left(\frac{180}{\pi}\right)$ degrees
$= 45$ degrees
$= 45^\circ$

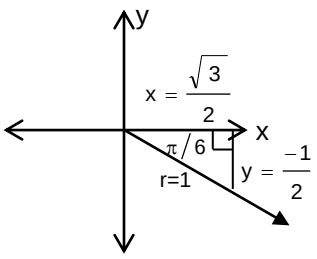
b) $\frac{-4}{5}\pi$ radians $= \frac{-4}{5}\pi(1 \text{ radian})$
$= \frac{-4\pi}{5}\left(\frac{180}{\pi}\right)$ degrees
$= \frac{-720}{5}$ degrees
$= -144^\circ$



Section 2.5 – Radian Measure of Angles (continued)

Example 4: Find the exact value of each of the following:

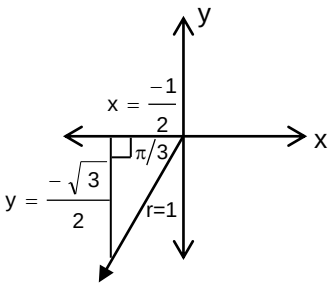
a) $\tan\frac{11\pi}{6}$



$$\theta_{ref} = 2\pi - \theta_c = \frac{12\pi}{6} - \frac{11\pi}{6} = \frac{\pi}{6}$$

$$\tan\frac{11\pi}{6} = \frac{y}{x} = \frac{\frac{-1}{2}}{\frac{\sqrt{3}}{2}} = \frac{-1}{\sqrt{3}} = \frac{-1}{\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right) = \frac{-\sqrt{3}}{3}$$

b) $\cos\frac{4\pi}{3}$

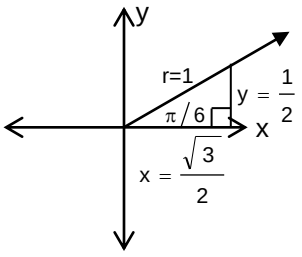


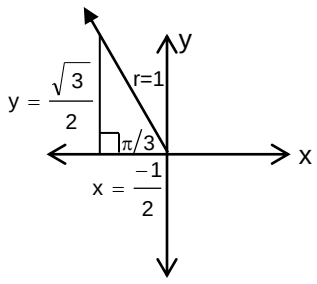
$$\theta_{ref} = \theta_c - \pi = \frac{4\pi}{3} - \pi = \frac{\pi}{3}$$

$$\cos\frac{4\pi}{3} = \frac{x}{r} = \frac{\frac{-1}{2}}{1} = \frac{-1}{2}$$

Example 5: Evaluate $3\sin\frac{\pi}{6} - 5\cot\frac{2\pi}{3}$.

$$3\sin\frac{\pi}{6} - 5\cot\frac{2\pi}{3} = 3\frac{y}{r} - 5\frac{x}{y}$$





$$\theta_{ref} = \pi - \theta_c = \pi - \frac{2\pi}{3} = \frac{\pi}{3}$$

$$= 3\left(\frac{\frac{1}{2}}{1}\right) - 5\left(\frac{\frac{-1}{2}}{\frac{\sqrt{3}}{2}}\right) = 3\left(\frac{1}{2}\right) - 5\left(\frac{-1}{\sqrt{3}}\right) = \frac{3}{2} + \frac{5}{\sqrt{3}} = \frac{3}{2} + \frac{5}{\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right) = \frac{3}{2} + \frac{5\sqrt{3}}{3}$$

All material has been taken from *Trigonometry,* by P. Foerster, 3rd Edition