Section 2.6 – Circular Functions and Inverses

In Section 1.2, we looked at some real-world situations in which the graph repeated itself at regular intervals. We then introduced the sine and cosine functions, which repeat themselves at regular intervals because the terminal side of the angle keeps coming back to the same position as it rotates around.

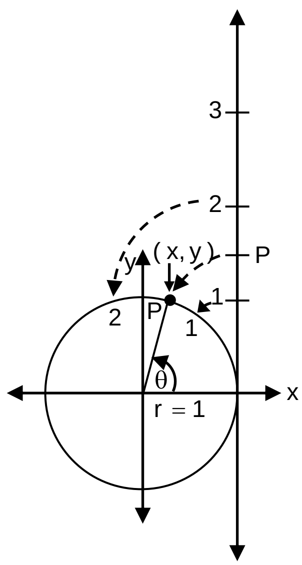
In many real-world problems, however, there is no apparent angle. This was true of the tide problem, the sunrise problem, and the breathing problem. So, we want to invent a function that behaves the same way as the trigonometric functions, but for which the independent variable is just a number rather than an angle.

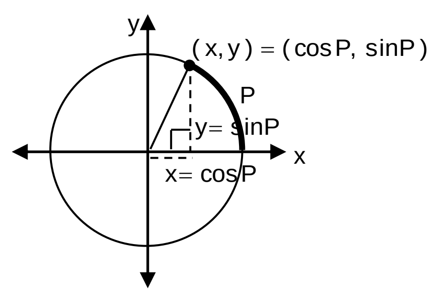
Objective: Define a set of functions that behaves like the trigonometric functions but whose independent variable is a number rather than an angle measure.

Numbers can be represented on a number line. As you saw in Section 2.5, a number line can be wrapped around a unit circle. The point P on the number line is thus represented by an arc of the circle that starts at the point $(1, 0)$ and ends at the point P. Let (x, y) be the coordinate of point P on the circle. If you draw an angle θ whose terminal side passes through (x, y), you can write the trigonometric functions of θ.

$$\sin\theta = \frac{y}{r} = \frac{y}{1} = y \qquad \text{and} \qquad \cos\theta = \frac{x}{r} = \frac{x}{1} = x$$

The point (x, y) is uniquely determined by both the angle θ and the number P. Therefore, it is natural to define the sine of the number P to be y also. So, $\sin P = y$ and $\cos P = x$. This new sine function is called a circular function, since its independent variable P equals the length of an arc on a unit circle. Similarly, the new cosine function is called a circular function.





Definition: If (x, y) is the terminal point of an arc of length P on a unit circle that starts at the point $(1, 0)$, then the circular sine and cosine are: $\sin P = y$ and $\cos P = x$.

The other circular functions are defined in terms of the sine and cosine.

Definition: The remaining circular functions are defined as follows:

$$\tan P = \frac{\sin P}{\cos P} \qquad \cot P = \frac{\cos P}{\sin P} \qquad \sec P = \frac{1}{\cos P} \qquad \csc P = \frac{1}{\sin P}$$

So, what does this mean for you? It means that if the independent variable is given as θ, then you are working with trigonometric functions and your angle will be in degrees. If the independent variable is given as x, then you are working with circular functions and your angle will be in radians. So, pay attention to the mode on your calculator.

All material has been taken from *Trigonometry,* by P. Foerster, 3rd Edition