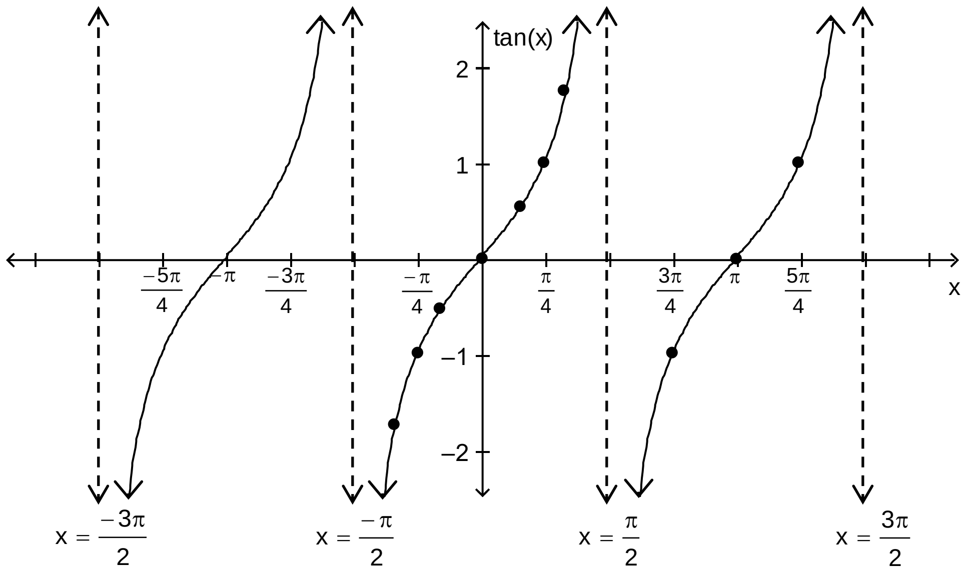
Section 2.9 – Graphs of Tangent, Cotangent, Secant, and Cosecant

Objective: Given an equation of the form $y = VD + A\,f\left[B(x - PD)\right]$, where f is tan, cot, sec, or csc, be able to sketch the graph.

The Graph of $f(x) = \tan x$

The tangent function has period π. The tangent is not defined at $\ldots, \frac{-3\pi}{2}, \frac{-\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \ldots,$ so we will look at the interval $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$. The rest of the graph will consist of repetitions of this portion of the graph.

x	$f(x) = \tan x$
$-\pi/2$	undefined
$-\pi/3$	$-\sqrt{3} \approx -1.73$
$-\pi/4$	-1
$-\pi/6$	$-\sqrt{3}/3 \approx -0.58$
0	0
$\pi/6$	$\sqrt{3}/3 \approx 0.58$
$\pi/4$	1
$\pi/3$	$\sqrt{3} \approx 1.73$
$\pi/2$	undefined



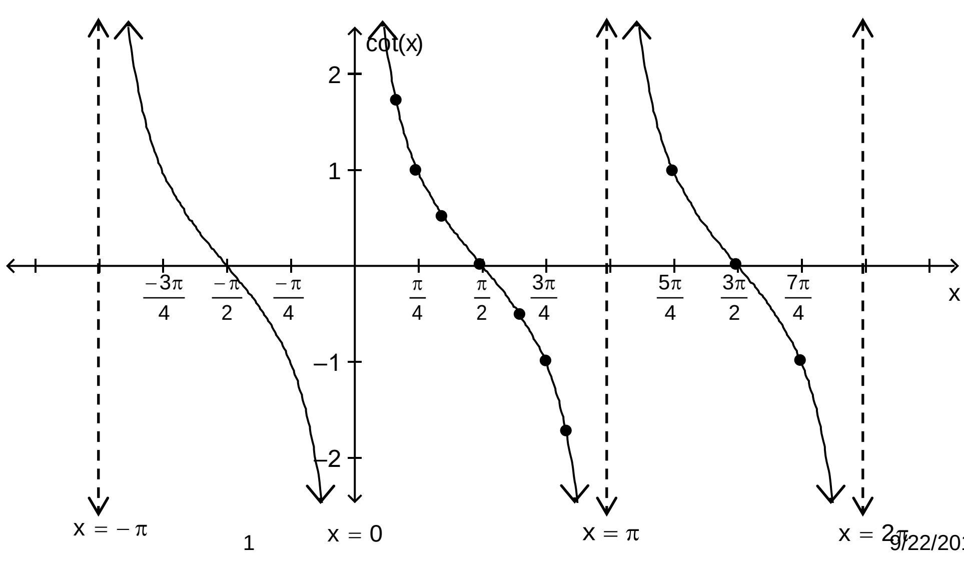
Characteristics of the Tangent Function:

1) The domain is the set of all real numbers, except odd multiples of $\frac{\pi}{2}$.
2) The range consists of all real numbers.
3) The tangent function is periodic, with period π.
4) The x-intercepts are $\ldots, -2\pi, -\pi, 0, \pi, 2\pi, 3\pi, \ldots,$ integral multiples of π. The y-intercept is 0.
5) Vertical asymptotes occur at $x = \ldots, \frac{-3\pi}{2}, \frac{-\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \ldots$.

The Graphs of $f(x) = \cot x$, $f(x) = \csc x$, and $f(x) = \sec x$.

The cotangent function has period π. The cotangent is not defined at integral multiples of π $(\ldots, -2\pi, -\pi, 0, \pi, 2\pi, \ldots)$, so we will look at the interval $(0, \pi)$. The rest of the graph will consist of repetitions of this portion of the graph.

x	$f(x) = \cot x$
0	undefined
$\pi/6$	$\sqrt{3} \approx 1.73$
$\pi/4$	1
$\pi/3$	$\sqrt{3}/3 \approx 0.58$
$\pi/2$	0
$2\pi/3$	$-\sqrt{3}/3 \approx -0.58$
$3\pi/4$	-1
$5\pi/6$	$-\sqrt{3} \approx -1.73$
π	undefined



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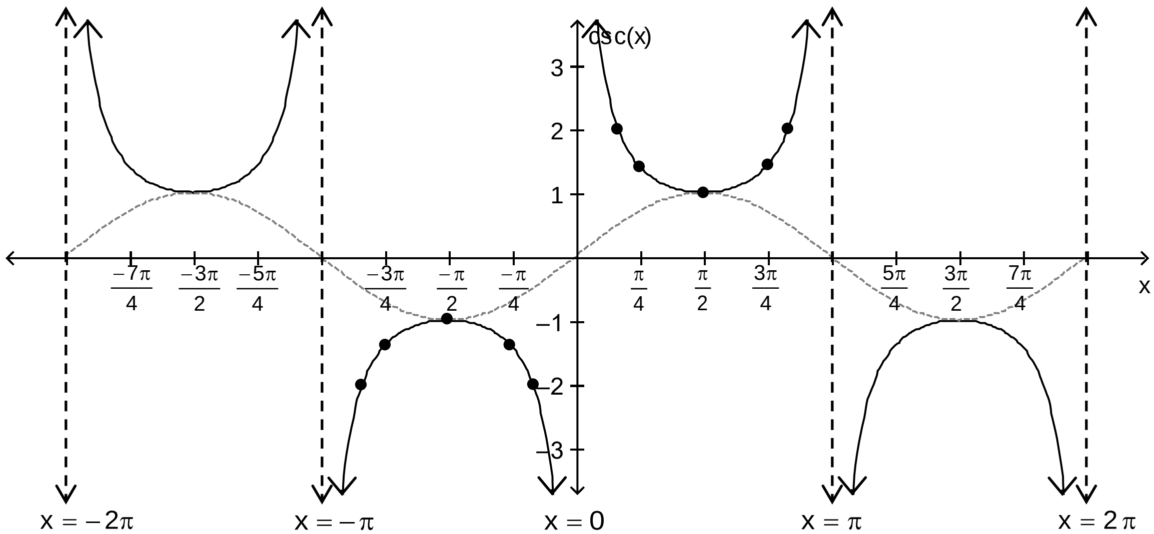
The cosecant and secant functions, referred to as reciprocal functions, are graphed using the reciprocal identities $\csc x = \dfrac{1}{\sin x}$ and $\sec x = \dfrac{1}{\cos x}$.

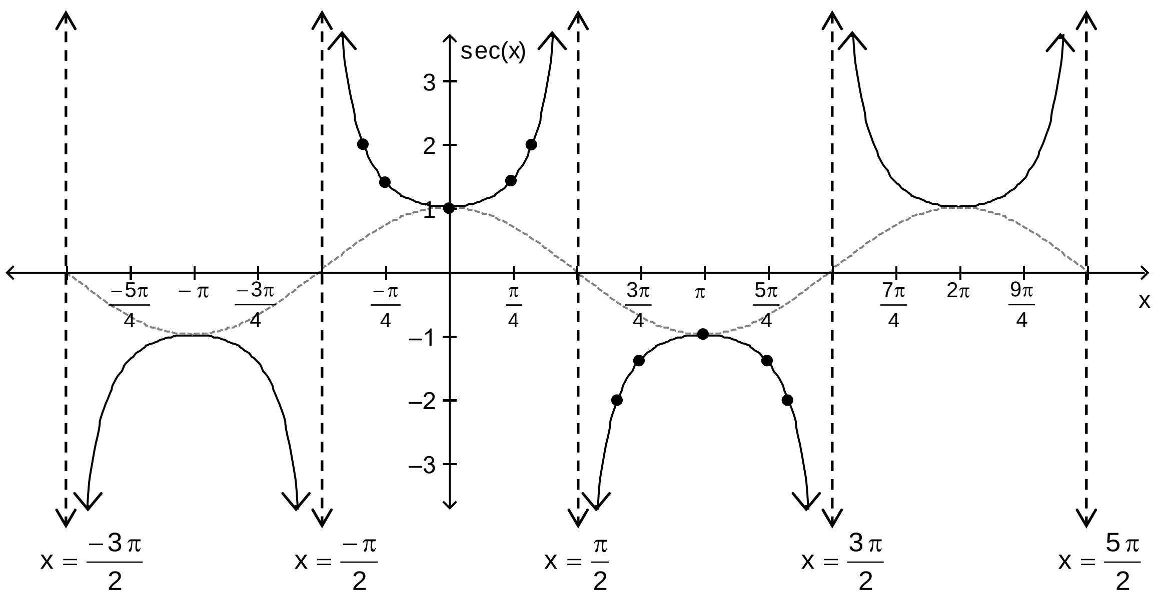
Since the sine function is 0 at integral multiples of π, the cosecant function has vertical asymptotes at integral multiples of π.

x	f(x)
$-5\pi/6$	-2
$-3\pi/4$	-1.41
$-\pi/2$	-1
$-\pi/4$	-2
$-\pi/6$	-1.41
$\pi/6$	2
$\pi/4$	1.41
$\pi/2$	1
$3\pi/4$	1.41
$5\pi/6$	2



Since the cosine function is 0 at odd multiples of $\dfrac{\pi}{2}$, the secant function has vertical asymptotes at odd multiples of $\dfrac{\pi}{2}$.

x	f(x)
$-\pi/3$	2
$-\pi/4$	1.41
0	1
$\pi/4$	1.41
$\pi/3$	2
$2\pi/3$	-2
$3\pi/4$	-1.41
π	-1
$5\pi/4$	-1.41
$4\pi/3$	-2



Once you learn the shapes of the graphs, you can plot them quickly by finding critical features. The most significant difference between these graphs and the sinusoidal graphs is that they are unbounded in the y-direction. They have vertical asymptotes at regular intervals because these functions can be written as ratios involving sines and cosines.

$$\tan x = \frac{\sin x}{\cos x}, \quad \cot x = \frac{\cos x}{\sin x}, \quad \sec x = \frac{1}{\cos x}, \text{ and } \csc x = \frac{1}{\sin x}.$$

Wherever the cosine function is zero, the tangent and secant functions will have asymptotes. Wherever the sine function is zero, the cotangent and cosecant functions will have asymptotes. Also note that between 0 and $\dfrac{\pi}{2}$ the functions (tangent and secant) are increasing and the cofunctions (cotangent and cosecant) are decreasing.

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For the basic functions $y = \tan x$ and $y = \cot x$, the period is π, so for tangent and cotangent use $\text{Period} = \dfrac{\pi}{B}$.

For the basic functions $y = \sec x$ and $y = \csc x$, the period is 2π, so for secant and cosecant use $\text{Period} = \dfrac{2\pi}{B}$.

Example 1: Sketch the graph of $y = 2 + 3\tan\left[\dfrac{\pi}{3}(x-2)\right]$

$y = VD + A\tan\left[B(x - PD)\right]$

$\Rightarrow \; A = 3,\; B = \dfrac{\pi}{3},\; VD = 2,$ and $PD = 2$

Phase displacement: $PD = 2 \;\Rightarrow$ shift right 2 units

Vertical displacement: $VD = 2 \;\Rightarrow$ shift up 2 units, so the sinusoidal axis is at $y = 2$

Locate and graph asymptotes:

The period of the basic tan is π, so $\text{Period} = \dfrac{\pi}{B}$

$$\text{Period} = \frac{\pi}{\frac{\pi}{3}}$$

$$\text{Period} = \pi\left(\frac{3}{\pi}\right)$$

$$\text{Period} = 3 \text{ units}$$

Since $\tan x = \dfrac{\sin x}{\cos x}$, $\tan x$ will have an asymptote wherever $\cos x = 0$. Now, $\cos x$ is 0 at all odd integral multiples of $\dfrac{\pi}{2}$. So, this tangent function will have asymptotes wherever its argument is equal to an odd integral multiple of $\dfrac{\pi}{2}$. Find one asymptote, then the others will be at that value plus or minus integral multiples of the period of the function.

So, set $\dfrac{\pi}{3}(x-2) = \dfrac{\pi}{2}$ and solve for x.

$$6\left(\frac{\pi}{3}\right)(x-2) = 6\left(\frac{\pi}{2}\right)$$

$$2\pi(x-2) = 3\pi$$

$$x - 2 = \frac{3\pi}{2\pi}$$

$$x - 2 = \frac{3}{2}$$

$$x = \frac{3}{2} + 2$$

$$x = \frac{3}{2} + \frac{4}{2}$$

$x = \dfrac{7}{2}$ This is one asymptote of the function. The other asymptotes will be at $x = \dfrac{7}{2} \pm$ (integral multiples of the period 3).

So, the asymptotes are at $\ldots,\; x = \dfrac{7}{2} - 2(3),\quad x = \dfrac{7}{2} - 3,\quad x = \dfrac{7}{2},\quad x = \dfrac{7}{2} + 3,\quad x = \dfrac{7}{2} + 2(3),\; \ldots$

$\Rightarrow \ldots,\; x = \dfrac{7}{2} - 6,\quad x = \dfrac{7}{2} - 3,\quad x = \dfrac{7}{2},\quad x = \dfrac{7}{2} + 3,\quad x = \dfrac{7}{2} + 6,\; \ldots$

$\Rightarrow \ldots,\; x = \dfrac{-5}{2},\quad x = \dfrac{1}{2},\quad x = \dfrac{7}{2},\quad x = \dfrac{13}{2},\quad x = \dfrac{19}{2},\; \ldots$

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Half-way between the asymptotes, the graph will cross the sinusoidal axis, so

$$\text{at } x_{midpt} = \frac{\frac{1}{2}+\frac{7}{2}}{2}$$

$$x_{midpt} = \frac{8/2}{2}$$

$$x_{midpt} = \frac{4}{2}$$

$$x_{midpt} = 2 \quad \Rightarrow \text{ the point } (x_{midpt}, VD) = (2, 2) \text{ is on the graph}$$

Half-way between the x_{midpt} and the right asymptote, the graph will be at the following point:

$$x = \frac{x_{midpt} + \frac{7}{2}}{2}$$

$$x = \frac{2+\frac{7}{2}}{2}$$

$$x = \frac{\frac{11}{2}}{2}$$

$$x = \frac{11}{2}\left(\frac{1}{2}\right)$$

$$x = \frac{11}{4}$$

The y-value is at $y = VD + 1(A)$

$$y = 2 + A$$

$$y = 2 + 3$$

$$y = 5$$

$\Rightarrow$ the point $(x, y) = \left(\frac{11}{4}, 5\right)$ is on the graph

Half-way between the x_{midpt} and the left asymptote, the graph will be at the following point:

$$x = \frac{\frac{1}{2} + x_{midpt}}{2}$$

$$x = \frac{\frac{1}{2}+2}{2}$$

$$x = \frac{\frac{5}{2}}{2}$$

$$x = \frac{5}{2}\left(\frac{1}{2}\right)$$

$$x = \frac{5}{4}$$

The y-value is at $y = VD - 1(A)$

$$y = 2 - A$$

$$y = 2 - 3$$

$$y = -1$$

$\Rightarrow$ the point $(x, y) = \left(\frac{5}{4}, -1\right)$ is on the graph

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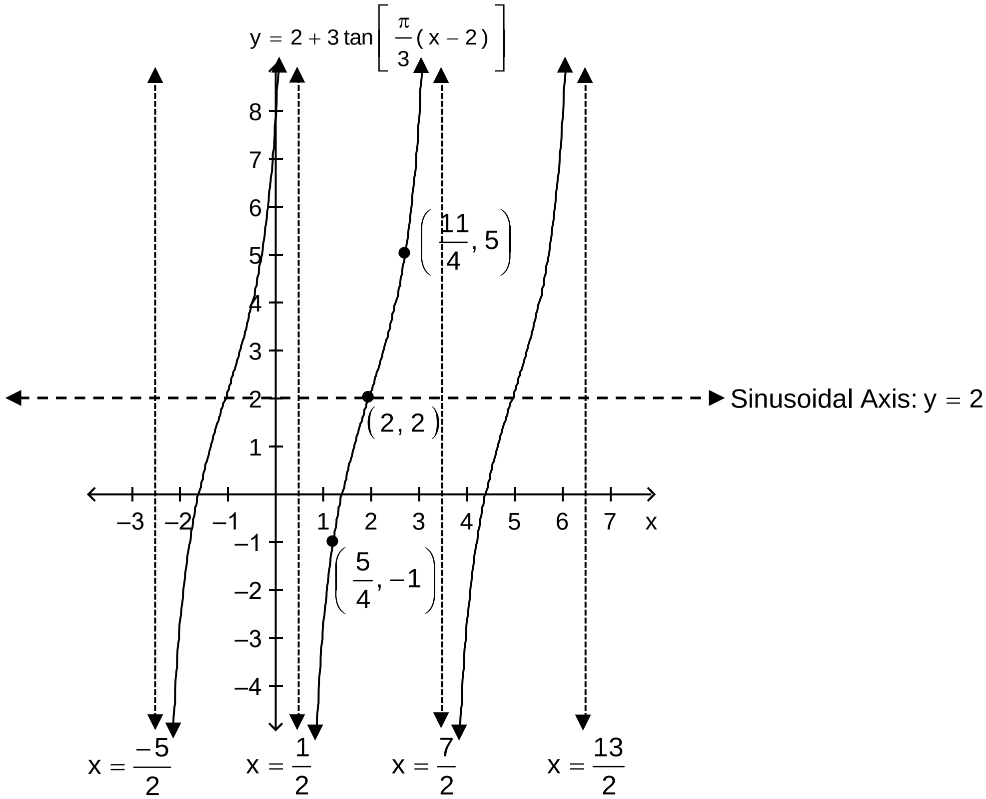

All material has been taken from ***Trigonometry,*** by P. Foerster, 3rd Edition