Section 2.11 – Evaluation of Sinusoidal Functions

Objectives: Given the equation of a sinusoid function,
1) find values of $f(x)$ for given values of x,
2) find values of x for given values of $f(x)$.

We have learned that the general equation for a sinusoidal function is
$f(x) = VD + A\sin[B(x - PD)]$ or $f(x) = VD + A\cos[B(x - PD)]$.

In the real world, you must be able to find values of $f(x)$ if x is given, and find values of x if $f(x)$ is given.

Example 1: Given $f(x) = 1 + 2\cos\left[\frac{\pi}{5}(x-3)\right]$

$f(x) = VD + A\cos[B(x - PD)]$

$\Rightarrow A = 2,\ B = \frac{\pi}{5},\ VD = 1,$ and $PD = 3$

Amplitude $= |A| = |2| = 2$

Period $P = \frac{2\pi}{B} = \frac{2\pi}{\frac{\pi}{5}} = 2\pi\left(\frac{5}{\pi}\right) = 10$

Phase displacement: $PD = 3 \Rightarrow$ shift right 3 units

Vertical displacement: $VD = 1 \Rightarrow$ shift up 1 unit, so the sinusoidal axis is at $y = 1$

a) Find $f(8)$.

$f(8) = 1 + 2\cos\left[\frac{\pi}{5}(8-3)\right]$

$= 1 + 2\cos\left[\frac{\pi}{5}(5)\right]$

$= 1 + 2\cos(\pi)$

$= 1 + 2\frac{x}{r}$ $\quad \pi = (-1, 0)$

$= 1 + 2\left(\frac{-1}{1}\right)$

$= 1 + 2(-1)$

$= 1 + -2$

$= -1$

b) Find $f(5)$.

$f(5) = 1 + 2\cos\left[\frac{\pi}{5}(5-3)\right]$

$= 1 + 2\cos\left[\frac{\pi}{5}(2)\right]$

$= 1 + 2\cos\left[\frac{2\pi}{5}\right]$

$\approx 1 + 2(0.3090)$

$= 1 + 0.6180$

$= 1.6180$

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c) Find x when $f(x) = 2$. Find the first three positive values of x.

$$f(x) = 2$$

$$1 + 2\cos\left[\frac{\pi}{5}(x-3)\right] = 2$$

$$2\cos\left[\frac{\pi}{5}(x-3)\right] = 1$$

$$\cos\left[\frac{\pi}{5}(x-3)\right] = \frac{1}{2}$$

$$\Rightarrow \left[\frac{\pi}{5}(x-3)\right] = \cos^{-1}\left(\frac{1}{2}\right)$$

$$x - 3 = \frac{5}{\pi}\cos^{-1}\left(\frac{1}{2}\right)$$

$$x = 3 + \frac{5}{\pi}\cos^{-1}\left(\frac{1}{2}\right)$$

$$x = 3 + \frac{5}{\pi}\left(\pm\frac{\pi}{3} + 2\pi n\right) \text{ for n any integer}$$

$$= 3 \pm \frac{5}{3} + 10n$$

If $n = -1$: x values are negative

If $n = 0$: $x = 3 + \frac{5}{3}$ or $x = 3 - \frac{5}{3}$

$$x = \frac{14}{3} \qquad x = \frac{4}{3}$$

$$\approx 4.6667 \qquad \approx 1.3333$$

If $n = 1$: $x = 3 + \frac{5}{3} + 10$ or $x = 3 - \frac{5}{3} + 10$

$$x = \frac{44}{3} \qquad x = \frac{34}{3}$$

$$\approx 14.6667 \qquad \approx 11.3333$$

So the first three positive values of x are $x = 1.3333,\ 4.6667,\ 11.3333$.

Example 2: Given $f(x) = -3 + 4\sin\left[\frac{\pi}{3}(x+9)\right]$

$$f(x) = VD + A\sin[B(x - PD)]$$

$$\Rightarrow\ A = 4,\ B = \frac{\pi}{3},\ VD = -3,\ \text{and}\ PD = -9$$

Amplitude $= |A| = |4| = 4$

Period $= \frac{2\pi}{B} = \frac{2\pi}{\frac{\pi}{3}} = 2\pi\left(\frac{3}{\pi}\right) = 6$

Phase displacement: $PD = -9 \Rightarrow$ shift left 9 units

Vertical displacement: $VD = -3 \Rightarrow$ shift down 3 units, so the sinusoidal axis is at $y = -3$

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b) Find $f(6)$.

$$f(6) = -3 + 4\sin\left[\frac{\pi}{3}(6+9)\right]$$
$$= -3 + 4\sin\left[\frac{\pi}{3}(15)\right]$$
$$= -3 + 4\sin(5\pi)$$
$$= -3 + 4\frac{y}{r} \qquad 5\pi = (-1, 0)$$
$$= -3 + 4\left(\frac{0}{1}\right)$$
$$= -3 + 4(0)$$
$$= -3 + 0$$
$$= -3$$

b) Find $f(-5)$.

$$f(-5) = -3 + 4\sin\left[\frac{\pi}{3}(-5+9)\right]$$
$$= -3 + 4\sin\left[\frac{\pi}{3}(4)\right]$$
$$= -3 + 4\sin\left[\frac{4\pi}{3}\right]$$
$$= -3 + 4\frac{y}{r} \qquad \frac{4\pi}{3} = \left(\frac{-1}{2}, \frac{-\sqrt{3}}{2}\right)$$
$$= -3 + 4\left(\frac{\frac{-\sqrt{3}}{2}}{1}\right)$$
$$= -3 + 4\left(\frac{-\sqrt{3}}{2}\right)$$
$$= -3 + -2\sqrt{3}$$
$$\approx -6.4641$$

c) Find x when $f(x) = -4$. Find the first three positive values of x.

$$f(x) = -4$$
$$-3 + 4\sin\left[\frac{\pi}{3}(x+9)\right] = -4$$
$$4\sin\left[\frac{\pi}{3}(x+9)\right] = -1$$
$$\sin\left[\frac{\pi}{3}(x+9)\right] = \frac{-1}{4}$$
$$\Rightarrow \left[\frac{\pi}{3}(x+9)\right] = \sin^{-1}\left(\frac{-1}{4}\right)$$
$$x + 9 = \frac{3}{\pi}\sin^{-1}\left(\frac{-1}{4}\right)$$
$$x = -9 + \frac{3}{\pi}\sin^{-1}\left(\frac{-1}{4}\right)$$
$$x = -9 + \frac{3}{\pi}(-0.2527 + 2\pi n) \quad \text{or} \quad x = -9 + \frac{3}{\pi}(\pi - (-0.2527) + 2\pi n) \text{ for n any integer}$$
$$x = -9 + \frac{3}{\pi}(\pi + 0.2527 + 2\pi n) \text{ for n any integer}$$
$$x \approx -9 + -0.2413 + 6n \quad \text{or} \quad x \approx -9 + 3 + 0.2413 + 6n \text{ for n any integer}$$
$$x \approx -9.2413 + 6n \quad \text{or} \quad x \approx -5.7587 + 6n \text{ for n any integer}$$

If $n = -1$ and $n = 0$: x values are negative

If $n = 1$: $x \approx -3.2413$ or $x \approx 0.2413$

If $n = 2$: $x \approx 2.7587$ or $x \approx 6.2413$

So the first three positive values of x are $x = 0.2413, 2.7587, 6.2413$.