

Binding Energy and Mass Defect

Worked Examples and questions

Data:

Particle	Relative Charge	Electric Charge (C)	Relative Mass (u)	Mass (kg)
Electron	-1	-1.60×10^{-19}	5.485779×10^{-4}	9.109390×10^{-31}
Proton	+1	$+1.60 \times 10^{-19}$	1.007276	1.672623×10^{-27}
Neutron	0	0	1.008665	1.674929×10^{-27}
$1u = 1.6605 \times 10^{-27} \text{ kg}$				
$1eV = 1.60 \times 10^{-19} \text{ Joules}$				
<i>The 'cheating' equivalence shortcut</i> $1u = 931.5 \text{ MeV}$				

Problem

${}^4_2\text{H}$ is the most abundant isotope of helium. Its mass is $6.6447 \times 10^{-27} \text{ kg}$. What is

- The mass defect?
- The binding energy of the nucleus in joules?
- The binding energy of the nucleus in electron volts?

Solution

a) Mass of component parts $m = 2p + 2n$
 $= 2(1.672623 \times 10^{-27}) + 2(1.674929 \times 10^{-27})$
 $m = 6.6950 \times 10^{-27} \text{ kg}$

Mass defect $= 6.6950 \times 10^{-27} \text{ kg} - 6.6447 \times 10^{-27} \text{ kg}$
 $= 5.03 \times 10^{-29} \text{ kg}$

b) Binding energy using $E = mc^2$
 $E = [5.03 \times 10^{-29} \text{ kg}] \times [3 \times 10^8]^2$
 $E = 4.53 \times 10^{-12} \text{ Joules}$

c) Binding energy $= 4.53 \times 10^{-12} \times (1/1.60 \times 10^{-19})$
 $= 2.83 \times 10^7 \text{ eV}$
 $[= 28.3 \text{ MeV }]$

Questions:

1) $^{238}_{92}\text{U}$ decays into $^{234}_{90}\text{Th}$ and an alpha particle

- Write down the full decay equation
- How much energy is released.

$$\text{Mass of } ^{238}_{92}\text{U} = 238.0508\text{u}$$

$$\text{Mass of } ^{234}_{90}\text{Th} = 234.0426\text{u}$$

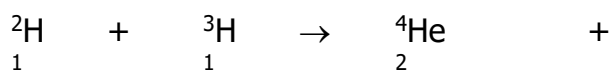
$$\text{Mass of } ^4_2\alpha = 4.0026\text{u}$$

2) Calculate the mass defect and binding energy the nuclide $^{10}_5\text{B}$ where the mass of $^{10}_5\text{B}$ atom = 10.0129 u

3) Oxygen has an unstable isotope O-17 that has a mass of 17.00454. If the mass of a neutron is 1.00898 u and the mass of a proton is 1.00814 u, calculate the binding energy of the oxygen nucleus in MeV.

4) A thorium atom of mass 232.038 u decays by the emission of an alpha particle to a radium atom of mass 228.031 u. If the alpha particle has a mass of 4.003 u, how much energy in J is released in the process ?

5) The fusion reaction below is one of the final stages in the fusion process that occurs in the Sun.



- Complete the reaction identifying the missing particle.
- Calculate the energy released in the fusion reaction using the following information (you will also need the mass of the other particle).

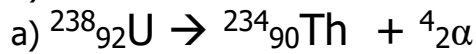
$$^2_1\text{H} = 3.345 \times 10^{-27} \text{ Kg}$$

$$^3_1\text{H} \rightarrow 5.008 \times 10^{-27} \text{ Kg}$$

$$^4_2\text{He} = 6.647 \times 10^{-27} \text{ Kg}$$

Solutions.

1)



b) First calculate mass change

$$238.0508\text{u} - (234.0426\text{u} + 4.0026\text{u})$$

$$\text{mass change} = 5.6 \times 10^{-3}\text{u}$$

$$\text{Convert to kg} = 5.6 \times 10^{-3}\text{u} \times 1.6605 \times 10^{-27}\text{kg}$$

$$\text{Mass defect} = 9.2988 \times 10^{-30}$$

$$\begin{aligned} \text{Energy released } E &= mc^2 \\ &= 9.2988 \times 10^{-30} \times (3 \times 10^8)^2 \\ &= 8.36892 \times 10^{-13} \text{ J} \end{aligned}$$

2) Calculate the mass defect and binding energy the nuclide ${}^{10}_5\text{B}$ where the mass of ${}^{10}_5\text{B}$ atom = 10.0129 u

${}^{10}_5\text{B}$ has 5 protons and 5 neutrons

$$\begin{aligned} \text{Total mass of nucleons} &= \text{mass of protons} + \text{mass of neutrons} \\ &= 5 [1.007276\text{u}] + 5 [1.008665\text{u}] \\ &= 5.03638\text{u} + 5.043325 \\ &= 10.079705\text{u} \end{aligned}$$

$$\begin{aligned} \text{Mass defect} &= \text{Mass of nucleons} - \text{mass of } {}^{10}_5\text{B nucleus} \\ &= 10.079705\text{u} - 10.0129 \text{ u} \\ &= 0.066805 \end{aligned}$$

$$\text{Mass defect in Kg} = 1.1093 \times 10^{-28} \text{ Kg}$$

$$\begin{aligned} \text{Binding Energy } E &= mc^2 \\ &= 1.1093 \times 10^{-28} \times (3 \times 10^8)^2 \\ &= 9.9836 \times 10^{-12} \text{ J} \end{aligned}$$

$$\begin{aligned} \text{Binding Energy in eV} &= 9.9836 \times 10^{-12} \text{ J} / 1.6 \times 10^{-19} \\ &= 6.2398 \times 10^7 \text{ eV} \\ &= 624 \text{ MeV} \end{aligned}$$

3) O-17 $^{17}_8\text{O}$ has 8 protons in the nucleus and 9 neutrons

$$\begin{aligned} \text{Total mass of nucleons} &= \text{mass of protons} + \text{mass of neutrons} \\ &= 8 [1.007276\text{u}] + 9 [1.008665\text{u}] \\ &= 8.058208\text{u} + 9.077985\text{u} \\ &= 17.136193\text{u} \end{aligned}$$

$$\begin{aligned} \text{Mass defect} &= \text{Mass of nucleons} - \text{mass of O17 nucleus} \\ &= 17.136193\text{u} - 17.00454\text{u} \\ &= 0.131653\text{u} \end{aligned}$$

$$\begin{aligned} \text{Mass defect in Kg} &= 0.131653 \times 1.6605 \times 10^{-27} \\ &= 2.186 \times 10^{-28} \text{ Kg} \end{aligned}$$

$$\begin{aligned} \text{Binding Energy } E &= mc^2 \\ &= 2.186 \times 10^{-28} \times (3 \times 10^8)^2 \\ &= 1.9675 \times 10^{-11} \text{ J} \end{aligned}$$

$$\begin{aligned} \text{Binding Energy in eV} &= 1.9675 \times 10^{-11} \text{ J} / 1.6 \times 10^{-19} \\ &= 1.2297 \times 10^8 \text{ eV} \\ &= 123 \text{ MeV} \end{aligned}$$

4) A thorium atom of mass 232.038 u decays by the emission of an alpha particle to a radium atom of mass 228.031 u. If the alpha particle has a mass of 4.003 u, how much energy in J is released in the process ?

Write out the reaction first (words will do here)



Calculate mass of products and reactants in terms of u

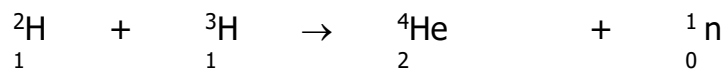
Reactants	Products
232.038u	228.031 + 4.003
232.038u	232.034

$$\begin{aligned} \text{Calculate the difference} &= 232.038 - 232.034 \\ &= 0.004\text{u} \end{aligned}$$

$$\begin{aligned} \text{Energy released } E &= mc^2 \\ &= 0.004 \times 1.66 \times 10^{-27} \times (3 \times 10^8)^2 \\ &= 5.976 \times 10^{-13} \text{ J} \end{aligned}$$

5)

(a)



(b) Calculate mass of products and reactants in Kg

Reactants

$$3.345 \times 10^{-27} + 5.008 \times 10^{-27} \text{ Kg}$$

$$8.353 \times 10^{-27}$$

Products

$$6.647 \times 10^{-27} \text{ Kg} + \text{mass of neutron}$$

$$6.647 \times 10^{-27} + 1.6605 \times 10^{-27} \times 1.008665$$

$$\begin{aligned} \text{Mass difference} &= 8.353 \times 10^{-27} - 8.321888 \times 10^{-27} \\ &= 3.1112 \times 10^{-29} \end{aligned}$$

$$\begin{aligned} \text{Energy released } E &= mc^2 \\ &= 3.1112 \times 10^{-29} \times (3 \times 10^8)^2 \\ &= 2.80 \times 10^{-12} \text{ J} \end{aligned}$$