Binding Energy and Mass Defect

Worked Examples and questions

Data:

Particle	Relative Charge	Electric Charge (C)	Relative Mass (u)	Mass (kg)
Electron	-1	-1.60×10^{-19}	5.485779×10^{-4}	9.109390×10^{-31}
Proton	+1	$+1.60 \times 10^{-19}$	1.007276	1.672623×10^{-27}
Neutron	0	0	1.008665	1.674929×10^{-27}
1u = 1.6605×10^{-27} kg				
1eV = 1.60×10^{-19} Joules				
The 'cheating' equivalence shortcut ***1u= 931.5 MeV***				

Problem

$^4_2$H is the most abundant isotope of helium. Its mass is 6.6447×10^{-27}kg. What is

a) The mass defect?

b) The binding energy of the nucleus in joules?

c) The binding energy of the nucleus in electron volts?

Solution

a) Mass of component parts m = 2p+2n

$= 2(1.672623 \times 10^{-27}) + 2(1.674929 \times 10^{-27})$

m = 6.6950×10^{-27}kg

Mass defect $= 6.6950 \times 10^{-27}$kg $- 6.6447 \times 10^{-27}$kg

$= 5.03 \times 10^{-29}$kg

b) Binding energy using E $= mc^2$

E $= [5.03 \times 10^{-29}\text{kg}] \times [3 \times 10^8]^2$

E = 4.53×10^{-12} Joules

c) Binding energy $= 4.53 \times 10^{-12} \times (1/1.60 \times 10^{-19})$

= 2.83×10^7 eV

[= 28.3 MeV]

Questions:

1) $^{238}_{92}U$ decays into $^{234}_{90}Th$ and an alpha particle

a) Write down the full decay equation

b) How much energy is released.

Mass of $^{238}_{92}U$	= 238.0508u
Mass of $^{234}_{90}Th$	= 234.0426u
Mass of $^{4}_{2}\alpha$	= 4.0026u

2) Calculate the mass defect and binding energy the nuclide $^{10}_{5}B$ where the mass of $^{10}_{5}B$ atom = 10.0129 u

3) Oxygen has an unstable isotope O-17 that has a mass of 17.00454. If the mass of a neutron is 1.00898 u and the mass of a proton is 1.00814 u, calculate the binding energy of the oxygen nucleus in MeV.

4) A thorium atom of mass 232.038 u decays by the emission of an alpha particle to a radium atom of mass 228.031 u. If the alpha particle has a mass of 4.003 u, how much energy in J is released in the process ?

5) The fusion reaction below is one of the final stages in the fusion process that occurs in the Sun.

$$^{2}_{1}H \quad + \quad ^{3}_{1}H \quad \rightarrow \quad ^{4}_{2}He \quad + $$

(a) Complete the reaction identifying the missing particle.

(b) Calculate the energy released in the fusion reaction using the following information (you will also need the mass of the other particle).

$^{2}_{1}H$ = 3.345×10^{-27} Kg

$^{3}_{1}H$ $\rightarrow$ 5.008×10^{-27} Kg

$^{4}_{2}He$ 6.647×10^{-27} Kg

Solutions.

1)

a) $^{238}_{92}U \rightarrow {}^{234}_{90}Th + {}^{4}_{2}\alpha$

b) First calculate mass change

238.0508u - (234.0426u + 4.0026u)
mass change = 5.6×10^{-3}u
Convert to kg = 5.6×10^{-3}u x 1.6605×10^{-27}kg

Mass defect = 9.2988×10^{-30}

Energy released	E	=	mc^2
		=	$9.2988 \times 10^{-30} \times (3\times10^8)^2$
		=	8.36892×10^{-13} J

2) Calculate the mass defect and binding energy the nuclide $^{10}_{5}B$ where the mass of $^{10}_{5}B$ atom = 10.0129 u

$^{10}_{5}B$ has 5 protons and 5 neutrons

Total mass of nucleons	=	mass of protons	+	mass of neutrons
	=	5 [1.007276u]	+	5 [1.008665u]
	=	5.03638u	+	5.043325
	=	10.079705u		
Mass defect	=	Mass of nucleons	-	mass of $^{10}_{5}B$ nucleus
	=	10.079705u	-	10.0129 u
	=	0.066805		
Mass defect in Kg	=	1.1093×10^{-28} Kg		
Binding Energy E	=	mc^2		
	=	$1.1093 \times 10^{-28} \times (3 \times 10^8)^2$		
	=	9.9836×10^{-12} J		
Binding Energy in eV	=	9.9836×10^{-12} J / 1.6×10^{-19}		
	=	6.2398×10^7 eV		
	=	624 MeV		

3) O-17 $^{17}_{8}O$ has 8 protons in the nucleus and 9 neutrons

Total mass of nucleons	=	mass of protons	+	mass of neutrons
	=	8 [1.007276u]	+	9 [1.008665u]
	=	8.058208u	+	9.077985u
	=	17.136193u		

Mass defect	=	Mass of nucleons - mass of O17 nucleus
	=	17.136193u - 17.00454u
	=	0.131653u
Mass defect in Kg	=	$0.131653 \times 1.6605 \times 10^{-27}$
	=	2.186×10^{-28} Kg
Binding Energy E	=	mc^2
	=	$2.186 \times 10^{-28} \times (3 \times 10^8)^2$
	=	1.9675×10^{-11} J
Binding Energy in eV	=	1.9675×10^{-11} J / 1.6×10^{-19}
	=	1.2297×10^8 eV
	=	123 MeV

4)A thorium atom of mass 232.038 u decays by the emission of an alpha particle to a radium atom of mass 228.031 u. If the alpha particle has a mass of 4.003 u, how much energy in J is released in the process ?

Write out the reaction first (words will do here)

Thorium $\rightarrow$ Radium + alpha particle

Calculate mass of products and reactants in terms of u

Reactants	Products
232.038u	228.031 + 4.003
232.038u	232.034

Calculate the difference	=	232.038 - 232.034
	=	0.004u
Energy released E	=	mc^2
	=	$0.004 \times 1.66 \times 10^{-27} \times (3 \times 10^8)^2$
	=	5.976×10^{-13} J

5)

(a)

$$^{2}_{1}H + ^{3}_{1}H \rightarrow ^{4}_{2}He + ^{1}_{0}n$$

(b) Calculate mass of products and reactants in Kg

Reactants

$3.345 \times 10^{-27} + 5.008 \times 10^{-27}$ Kg

8.353×10^{-27}

Products

6.647×10^{-27} Kg + mass of neutron

$6.647 \times 10^{-27} + 1.6605 \times 10^{-27} \times 1.008665$

Mass difference $= 8.353 \times 10^{-27} - 8.321888 \times 10^{-27}$

$= 3.1112 \times 10^{-29}$

Energy released $E = mc^2$

$= 3.1112 \times 10^{-29} \times (3 \times 10^{8})^2$

$= 2.80 \times 10^{-12}$ J