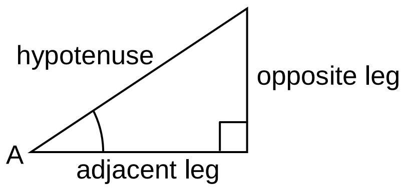
Section 5.1 – Right Triangle Review

Objective: Given the measures of two sides or a side and an angle in a right triangle, find the measures of the other side(s) and angle(s).

The word trigonometry comes from Greek words that mean "triangle measurement." In the right triangle below, one acute angle is named A. The leg opposite angle A and the leg adjacent to angle A are labeled. Adjacent means "next to" and here opposite means "across from."



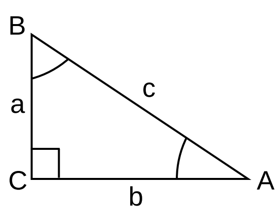
The definitions of the six trigonometric functions for angle A of a right triangle are as follows:

$$\sin A = \frac{\text{leg opposite } \angle A}{\text{hypotenuse}} \quad \text{or} \quad = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \csc A = \frac{\text{hypotenuse}}{\text{leg opposite } \angle A} \quad \text{or} \quad = \frac{\text{hypotenuse}}{\text{opposite}}$$

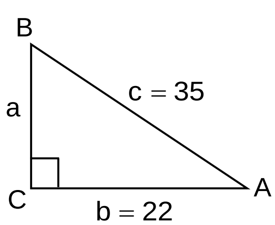
$$\cos A = \frac{\text{leg adjacent to } \angle A}{\text{hypotenuse}} \quad \text{or} \quad = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \sec A = \frac{\text{hypotenuse}}{\text{leg adjacent to } \angle A} \quad \text{or} \quad = \frac{\text{hypotenuse}}{\text{adjacent}}$$

$$\tan A = \frac{\text{leg opposite } \angle A}{\text{leg adjacent to } \angle A} \quad \text{or} \quad = \frac{\text{opposite}}{\text{adjacent}} \qquad \cot A = \frac{\text{leg adjacent to } \angle A}{\text{leg opposite } \angle A} \quad \text{or} \quad = \frac{\text{adjacent}}{\text{opposite}}$$

These definitions allow you to write the trigonometric functions of angle A even if the triangle is flipped over and rotated. The name of a triangle is usually the letters of the vertices. So the triangle below is called ΔABC. It is customary to name the length of the side opposite an angle with the same lowercase letter as the name of the angle. So, for example, the side opposite angle A has measure *a*.



Example 1: Find the six trigonometric functions for angle A in ΔABC.

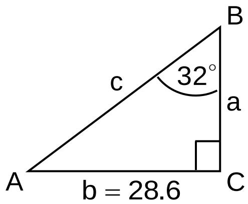


$$\sin A = \frac{\text{opp}}{\text{hyp}} = \frac{a}{35} \qquad \cos A = \frac{\text{adj}}{\text{hyp}} = \frac{22}{35} \qquad \tan A = \frac{\text{opp}}{\text{adj}} = \frac{a}{22}$$

$$\csc A = \frac{\text{hyp}}{\text{opp}} = \frac{35}{a} \qquad \sec A = \frac{\text{hyp}}{\text{adj}} = \frac{35}{22} \qquad \cot A = \frac{\text{adj}}{\text{opp}} = \frac{22}{a}$$

Section 5.1 – Right Triangle Review (continued)

Example 2: Find the missing side and angle measures for the following right triangle.
Given: In ΔABC, C is the right angle, $m\angle B = 32^\circ$, and $b = 28.6$.



$$m\angle C = 90^\circ$$
$$m\angle A = 90^\circ - m\angle B$$
$$= 90^\circ - 32^\circ$$
$$= 58^\circ$$

Using sin, cos, and tan only:

$$\tan B = \frac{opp}{adj}$$
$$\tan B = \frac{b}{a}$$
$$a\tan B = b$$
$$a = \frac{b}{\tan B}$$
$$a = \frac{28.6}{\tan 32^\circ}$$
$$a \approx 45.769$$
$$a \approx 45.77$$

$$\sin B = \frac{opp}{hyp}$$
$$\sin B = \frac{b}{c}$$
$$c\sin B = b$$
$$c = \frac{b}{\sin B}$$
$$c = \frac{28.6}{\sin 32^\circ}$$
$$c \approx 53.970$$
$$c \approx 53.97$$

OR
Solving with variables in the numerator:

$$\cot B = \frac{adj}{opp}$$
$$\cot B = \frac{a}{b}$$
$$a = b\cot B$$
$$a = \frac{b}{\tan B}$$
$$a = \frac{28.6}{\tan 32^\circ}$$
$$a \approx 45.769$$
$$a \approx 45.77$$

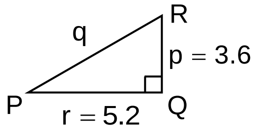
$$\csc B = \frac{hyp}{opp}$$
$$\csc B = \frac{c}{b}$$
$$c = b\csc B$$
$$c = \frac{b}{\sin B}$$
$$c = \frac{28.6}{\sin 32^\circ}$$
$$c \approx 53.970$$
$$c \approx 53.97$$

So, $m\angle A = 58^\circ$, $m\angle C = 90^\circ$, $a \approx 45.77$, and $c \approx 53.97$.

Check: $m\angle A + m\angle B + m\angle C = 58^\circ + 32^\circ + 90^\circ$
$= 180^\circ$

$a^2 + b^2 = c^2$?
$45.77^2 + 28.6^2 = 53.97^2$
$2094.8929 + 817.96 = 2912.7609$
$2912.8529 \approx 2912.7609$

Example 3: Find the missing side and angle measures for the following right triangle.
Given: In ΔPQR, Q is the right angle, $p = 3.6$, and $r = 5.2$.



$$m\angle Q = 90^\circ$$

$$\tan P = \frac{opp}{adj}$$
$$\tan P = \frac{p}{r}$$
$$\tan P = \frac{3.6}{5.2}$$
$$m\angle P = \tan^{-1}\left(\frac{3.6}{5.2}\right)$$
$$m\angle P \approx 34.69^\circ$$
$$m\angle P \approx 34.7^\circ$$

$$\tan R = \frac{opp}{adj}$$
$$\tan R = \frac{r}{p}$$
$$\tan R = \frac{5.2}{3.6}$$
$$m\angle R = \tan^{-1}\left(\frac{5.2}{3.6}\right)$$
$$m\angle R \approx 55.30^\circ$$
$$m\angle R \approx 55.3^\circ$$

$$\sin P = \frac{opp}{hyp}$$
$$\sin P = \frac{p}{q}$$
$$q\sin P = p$$
$$q = \frac{p}{\sin P}$$
$$q = \frac{3.6}{\sin 34.7^\circ}$$
$$q \approx 6.323$$
$$q \approx 6.32$$

OR
Solving with q in the numerator:

$$\csc P = \frac{hyp}{opp}$$
$$\csc P = \frac{q}{p}$$
$$p\csc P = q$$
$$q = \frac{p}{\sin P}$$
$$q = \frac{3.6}{\sin 34.7^\circ}$$
$$q \approx 6.323$$
$$q \approx 6.32$$

So, $m\angle P = 34.7^\circ$, $m\angle R = 55.3^\circ$, and $q \approx 6.32$.

Check: $m\angle P + m\angle Q + m\angle R = 34.7^\circ + 90^\circ + 55.3^\circ$
$= 180^\circ$

$p^2 + r^2 = q^2$?
$3.6^2 + 5.2^2 = 6.32^2$
$12.96 + 27.04 = 39.9424$
$40 \approx 39.9424$

All material has been taken from *Trigonometry,* by P. Foerster, 3rd Edition