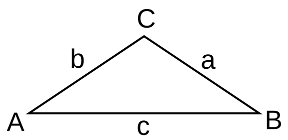
## Section 5.3 – Area of a Triangle

**Objectives**: 1. Given the measures of two sides and the included angle, find the area of the triangle.
2. Given the measures of three sides, find the area of the triangle.

In this section, we will look at formulas for calculating the area of a triangle.

We will always label an oblique triangle so that side *a* is opposite angle A, side b is opposite angle B, and side c is opposite angle C.




In geometry, you learned the following theorem about the area of a triangle.

Theorem:

The area of a triangle is $\text{Area} = \frac{1}{2}bh$, where b is the base of the triangle and h is an altitude drawn to that base.

But, if the base b and altitude h are not given, what do you do?

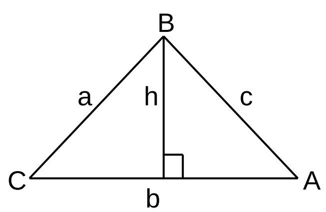
Theorem:

The area of a triangle equals one-half the product of two of its sides times the sine of their included angle.

So, we have the formulas:

$\text{Area} = \frac{1}{2}ab\sin C$ $\quad\quad$ $\text{Area} = \frac{1}{2}bc\sin A$ $\quad\quad$ $\text{Area} = \frac{1}{2}ac\sin B$

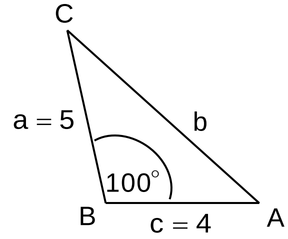
Why is this true? If we know two sides a and b and the included angle C, then we can find the altitude h.




$$\sin C = \frac{\text{opp}}{\text{hyp}}$$
$$\sin C = \frac{h}{a}$$
$$h = a\sin C$$
$$\begin{aligned}\text{Area} &= \frac{1}{2}bh\\ &= \frac{1}{2}b(a\sin C)\\ &= \frac{1}{2}ab\sin C\end{aligned}$$

Example 1: Find the area of the triangle with $a = 5$, $c = 4$, and $m\angle B = 100^\circ$.

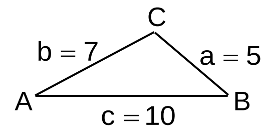
$$\begin{aligned}\text{Area} &= \frac{1}{2}ac\sin B\\ &= \frac{1}{2}(5)(4)\sin 100^\circ\\ &= \frac{20}{2}\sin 100^\circ\\ &= 10\sin 100^\circ\\ &\approx 9.848078\\ &\approx 9.85 \text{ square units}\end{aligned}$$



## Section 5.3 – Area of a Triangle (continued)

Example 2: Find the area of the triangle with $a = 5$, $b = 7$, and $c = 10$.

We need an angle to use the above theorem. So, use the Law of Cosines to find angle A.




$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$2bc\cos A = b^2 + c^2 - a^2$$
$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$
$$= \frac{7^2 + 10^2 - 5^2}{2(7)(10)}$$
$$= \frac{49 + 100 - 25}{140}$$
$$= \frac{124}{140}$$
$$\approx 0.885714$$
$$m\angle A = \cos^{-1}(0.885714)$$
$$m\angle A \approx 27.66^\circ$$
$$m\angle A \approx 27.7^\circ$$

So, $\text{Area} = \frac{1}{2}bc\sin A$
$$= \frac{1}{2}(7)(10)\sin 27.7^\circ$$
$$= \frac{1}{2}(70)\sin 27.7^\circ$$
$$= 35\sin 27.7^\circ$$
$$\approx 16.269$$

Area $\approx 16.27$ square units

The above area theorem is usually used to find the area of SAS triangles. To find the area of SSS triangles you can use the following theorem.

Theorem: Heron's Formula (sometimes known as Hero's Formula)

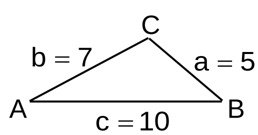
The area of a triangle with sides a, b, and c is

$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{1}{2}(a+b+c)$. ( s is called the "semiperimeter," half the perimeter of the triangle.)

(Due to Heron of Alexandria, about 75 AD)

Example 3: Find the area of a triangle whose sides are 5, 7, and 10. (This is the same problem done in Example 2, but you're solving it another way.)

Let $a = 5$, $b = 7$, and $c = 10$.




Then $s = \frac{1}{2}(a+b+c)$
$$= \frac{1}{2}(5+7+10)$$
$$= \frac{1}{2}(22)$$
$$= 11$$

The Area $= \sqrt{s(s-a)(s-b)(s-c)}$
$$= \sqrt{11(11-5)(11-7)(11-10)}$$
$$= \sqrt{11(6)(4)(1)}$$
$$= \sqrt{66(4)}$$
$$= \sqrt{264}$$
$$\approx 16.248$$
$\approx 16.25$ square units (Approximately the same answer as in Example 2: different due to rounding errors)

All material has been taken from *Trigonometry,* by P. Foerster, 3rd Edition