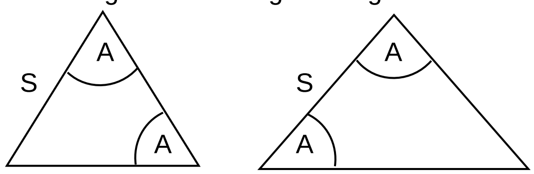
Section 5.4 – Oblique Triangles – Law of Sines

Objectives: 1. Given the measure of an angle, its opposite side, and one other angle measure, find the length of another side.
2. Given a side and the measures of the two angles at either end of this side, find the length of another side.

The Law of Cosines may be used for oblique triangles when you know two sides and the included angle or if you know all three sides. The Law of Sines will be used to find the measures of missing angles and sides in all remaining triangle configurations. In this section, we will learn how to find the missing sides and angle for oblique triangles in which one side and two angles are known (SAA or ASA).

The following are the triangle configurations in which one side and two angles are known. (SAA or ASA)



The Law of Sines is used to solve **SAA** or **ASA** triangles.

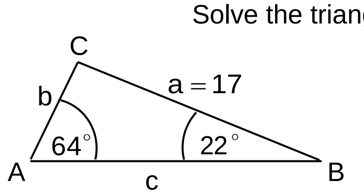
Theorem: Law of Sines

For a triangle with sides a, b, c and opposite angles A, B, and C, respectively, $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$.

We also use the fact that the sum of the angles of any triangle equals 180°, i.e., $m\angle A + m\angle B + m\angle C = 180^\circ$. So, given SAA or ASA information, determine the third angle first, then use the Law of Sines to find the missing sides.

Example 1: Case: SAA

Solve the triangle: $m\angle A = 64^\circ$, $m\angle B = 22^\circ$, $a = 17$.



$$m\angle A + m\angle B + m\angle C = 180^\circ$$
$$m\angle C = 180^\circ - m\angle A - m\angle B$$
$$m\angle C = 180^\circ - 64^\circ - 22^\circ$$
$$m\angle C = 180^\circ - 86^\circ$$
$$\underline{m\angle C = 94^\circ}$$

Using the Law of Sines,

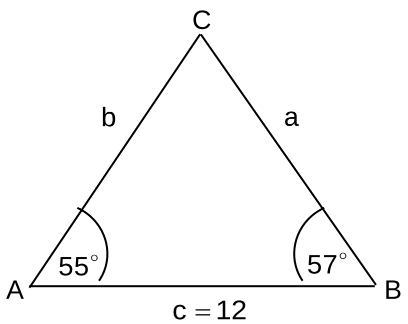
$$\frac{\sin A}{a} = \frac{\sin B}{b} \qquad\qquad \frac{\sin A}{a} = \frac{\sin C}{c}$$
$$b \sin A = a \sin B \qquad\qquad c \sin A = a \sin C$$
$$b = \frac{a \sin B}{\sin A} \qquad\qquad c = \frac{a \sin C}{\sin A}$$
$$\Rightarrow\ b = \frac{17 \sin 22^\circ}{\sin 64^\circ} \qquad\qquad \Rightarrow\ c = \frac{17 \sin 94^\circ}{\sin 64^\circ}$$
$$b \approx 7.085 \qquad\qquad c \approx 18.868$$
$$\underline{b \approx 7.09} \qquad\qquad \underline{c \approx 18.87}$$

In Example 1, the given side *a* was used to find sides b **and** c. Don't find side b and then use it to find side c, since b is a rounded value and/or you may have calculated b incorrectly. Whenever possible, always use given information.

Section 5.4 – Oblique Triangles – Law of Sines (continued)

Example 2: Case: ASA

Solve the triangle: $m\angle A = 55^\circ$, $m\angle B = 57^\circ$, $c = 12$.



$$m\angle A + m\angle B + m\angle C = 180^\circ$$
$$m\angle C = 180^\circ - m\angle A - m\angle B$$
$$m\angle C = 180^\circ - 55^\circ - 57^\circ$$
$$m\angle C = 180^\circ - 112^\circ$$
$$\underline{m\angle C = 68^\circ}$$

Using the Law of Sines,

$$\frac{\sin A}{a} = \frac{\sin C}{c} \qquad\qquad \frac{\sin B}{b} = \frac{\sin C}{c}$$
$$a\sin C = c\sin A \qquad\qquad b\sin C = c\sin B$$
$$a = \frac{c\sin A}{\sin C} \qquad\qquad b = \frac{c\sin B}{\sin C}$$
$$\Rightarrow \quad a = \frac{12\sin 55^\circ}{\sin 68^\circ} \qquad\qquad \Rightarrow \quad b = \frac{12\sin 57^\circ}{\sin 68^\circ}$$
$$a \approx 10.601 \qquad\qquad b \approx 10.854$$
$$\underline{a \approx 10.60} \qquad\qquad \underline{b \approx 10.85}$$

Where does the Law of Sines come from?

From what we did earlier in Section 5.3 on the areas of triangles, we know the following formulas for finding the area of an oblique triangle ABC.

$$\text{Area} = \frac{1}{2}bc\sin A \qquad \text{Area} = \frac{1}{2}ac\sin B \qquad \text{Area} = \frac{1}{2}ab\sin C$$

Since these are all area formulas for the same triangle, they must all be equal.

$$\text{Area} = \text{Area} = \text{Area}$$
$$\frac{1}{2}bc\sin A = \frac{1}{2}ac\sin B = \frac{1}{2}ab\sin C$$
$$bc\sin A = ac\sin B = ab\sin C$$
$$\frac{bc\sin A}{abc} = \frac{ac\sin B}{abc} = \frac{ab\sin C}{abc}$$
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c},$$ which is the Law of Sines

All material has been taken from *Trigonometry,* by P. Foerster, 3rd Edition