Section 5.6 – General Solution of Triangles

Objective: Given a SSS, SAS, ASA, AAS, or SSA triangle, be able to select the appropriate technique to calculate the remaining side and angle measures and the area of the triangle.

You have learned all the techniques necessary for analyzing oblique triangles. So far, you have been told which technique to use to solve a triangle. In this section, you must determine which technique to use to find the missing side and angle measures and the area of a triangle.

The following is a list of guidelines for determining which technique to use.

Triangle Techniques:

1) The Law of Cosines involves three sides. Therefore, it will not work for ASA or AAS, where there are two unknown sides.

2) The Law of Sines involves the ratio of the sine of an angle to the length of its opposite side. Therefore, it will not work where no angle is known (SSS) or where only one angle is known, but not its opposite side (SAS).

3) The Law of Sines should not be used to find angle measures unless you know in advance whether the angle is obtuse or acute.

4) The area formula $\left(\text{Area} = \frac{1}{2}ab\sin C\right)$ requires you to know SAS. If you do not know two sides and the included angle, you must first find them. If you know SSS, then you may use Heron's Formula to determine the area.

Use the Law of Cosines when you know at least two sides (SSS, SAS, SSA).

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$

To find an angle, say angle A: $a^2 = b^2 + c^2 - 2bc\cos A$

$$2bc\cos A = b^2 + c^2 - a^2$$
$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$
$$\text{So, } m\angle A = \cos^{-1}\left(\frac{b^2 + c^2 - a^2}{2bc}\right)$$

Use the Law of Sines when you know at least two angles (ASA, AAS).

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

To find the area of a triangle:

Use $\text{Area} = \frac{1}{2}ab\sin C$ for SAS

Use Heron's Formula for SSS:

The area of a triangle with sides a, b, and c is

$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{1}{2}(a+b+c)$. (s is called the "semiperimeter," half the perimeter of the triangle.)

Also remember that the three angles of a triangle add up to 180°.

$$m\angle A + m\angle B + m\angle C = 180^\circ$$

All material has been taken from *Trigonometry,* by P. Foerster, 3rd Edition