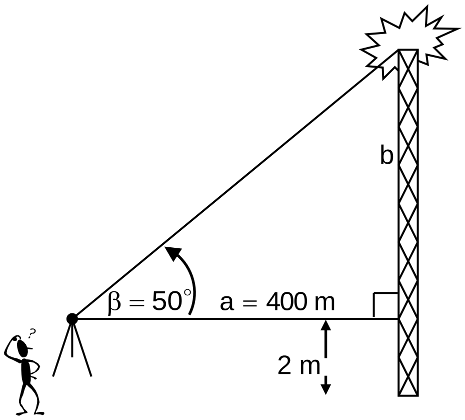
Section 5.9 – Real-World Triangle Problems

Throughout this chapter you have been developing the computational skills you need to work real-world problems involving measurement of triangles. Each real-world problem will require you to identify one or more triangles, right or oblique, and then apply the appropriate technique to find the side, angle, or area you seek.

Example 1: At a distance of 400 m from the base of a radio tower, the angle of elevation is measured and found to be 50°. If the transit is 2 m off the ground when the sighting is taken, how high is the radio tower?

A right triangle – use right triangle trigonometry



$$\tan\beta = \frac{\text{opp}}{\text{adj}}$$

$$\tan\beta = \frac{b}{a}$$

$$b = a\tan\beta$$

$$b = 400\tan 50^\circ$$

$$b \approx 476.701$$

$$b \approx 476.70\text{ m}$$

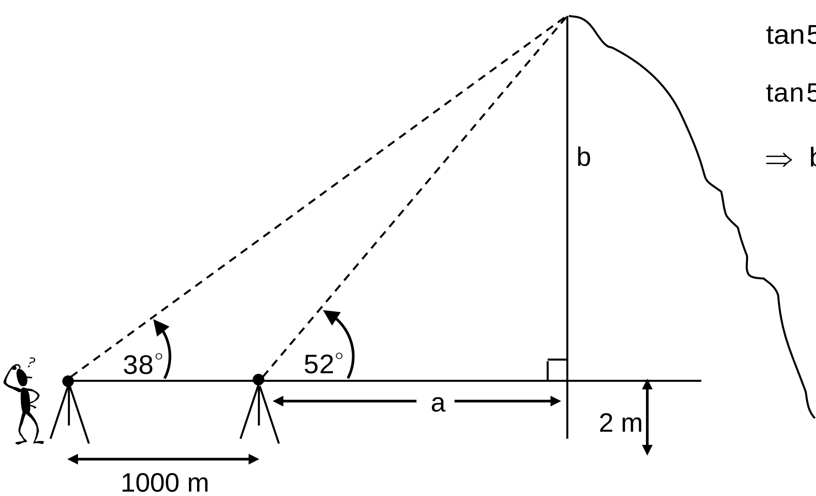
Height of the Tower $= 2 + b$

$= 2 + 476.70\text{ m}$

$= 478.70\text{ m}$

Example 2: To measure the height of a mountain, a surveyor takes two sightings of the peak at a distance of 1000 m apart. Assume these sightings are on a direct line to the mountain, and the sightings are taken from the same height. The first sighting results in an angle of elevation of 52° and the second sighting in an angle of elevation of 38°. If the transit is 2 m high, what is the height of the mountain?

A right triangle – use right triangle trigonometry



$\tan 52^\circ = \text{opp/adj}$ $\quad$ $\tan 38^\circ = \text{opp/adj}$

$\tan 52^\circ = \dfrac{b}{a}$ and $\tan 38^\circ = \dfrac{b}{a+1000}$

$\Rightarrow b = a\tan 52^\circ$ and $b = (a+1000)\tan 38^\circ$

Set b = b:

$$a\tan 52^\circ = (a+1000)\tan 38^\circ$$

$$a\tan 52^\circ = a\tan 38^\circ + 1000\tan 38^\circ$$

$$a\tan 52^\circ - a\tan 38^\circ = 1000\tan 38^\circ$$

$$a(\tan 52^\circ - \tan 38^\circ) = 1000\tan 38^\circ$$

$$a = \frac{1000\tan 38^\circ}{\tan 52^\circ - \tan 38^\circ}$$

$$a \approx 1566.782747\text{ m}$$

Then, $b = a\tan 52^\circ$

$\approx 1566.782747\tan 52^\circ$

≈ 2005.390467

$\approx 2005.39\text{ m}$

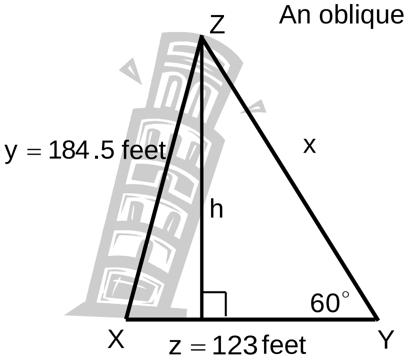
Thus, the height H of the mountain is approximately $H = b + 2$

$= 2005.39 + 2$

$= 2007.39\text{ m}$

Section 5.9 – Real-World Triangle Problems (continued)

Example 3: The Leaning Tower of Pisa was originally 184.5 feet high. At a distance of 123 feet from the base of the tower, the angle of elevation to the top of the tower is found to be 60°. Find the angle ZXY indicated in the figure. Also, find the perpendicular distance h from Z to $\overline{XY}$.



An oblique triangle

SSA: $y^2 = x^2 + z^2 - 2xz\cos Y$ Law of Cosines

$$184.5^2 = x^2 + 123^2 - 2x(123)\cos 60^\circ$$
$$34040.25 = x^2 + 15129 - 246\cos 60^\circ x$$
$$0 = x^2 - 246\cos 60^\circ x - 18911.25$$
$$0 = x^2 - 123x - 18911.25 \quad \text{A quadratic equation in the variable x}$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-123) \pm \sqrt{(-123)^2 - 4(1)(-18911.25)}}{2(1)}$$
$$x = \frac{123 \pm \sqrt{15129 + 75645}}{2}$$
$$x = \frac{123 \pm \sqrt{90774}}{2}$$
$$x = \frac{123 \pm 301.287238}{2}$$
$$x = \frac{123 + 301.287238}{2} \quad \text{or} \quad x = \frac{123 - 301.287238}{2}$$
$$x = \frac{424.287238}{2} \quad \text{or} \quad x = \frac{-178.287238}{2}$$
$$x = 212.143619 \quad \text{or} \quad x = -89.143619 \quad \text{Side length x cannot be negative}$$
$\Rightarrow$ One triangle with $x \approx 212.14$ feet

Find $m\angle ZXY \equiv m\angle X$:

$$x^2 = y^2 + z^2 - 2yz\cos X$$
$$2yz\cos X = y^2 + z^2 - x^2$$
$$\cos X = \frac{y^2 + z^2 - x^2}{2yz}$$
$$\cos X = \frac{184.5^2 + 123^2 - 212.14^2}{2(184.5)(123)}$$
$$\cos X = \frac{34040.25 + 15129 - 45003.3796}{2(22693.5)}$$
$$\cos X = \frac{4165.8704}{45387}$$
$$m\angle X = \cos^{-1}\left(\frac{4165.8704}{45387}\right)$$
$$m\angle X \approx \cos^{-1}(0.091786)$$
$$m\angle X \approx 84.73^\circ$$
$$m\angle X \approx 84.7^\circ$$
So, $m\angle ZXY \approx 84.7^\circ$

Find the perpendicular distance h from Z to $\overline{XY}$:

$$\frac{\sin Y}{h} = \frac{\sin 90^\circ}{x}$$
$$h\sin 90^\circ = x\sin Y$$
$$h = \frac{x\sin Y}{\sin 90^\circ}$$
$$h = \frac{212.14\sin 60^\circ}{\sin 90^\circ}$$
$$h \approx \frac{183.718629}{1}$$
$$h \approx 183.72 \text{ feet}$$