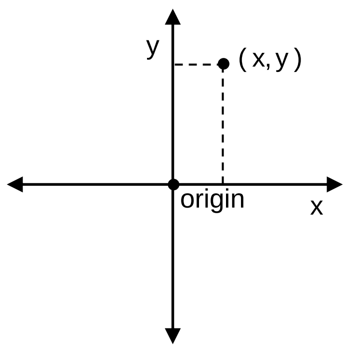
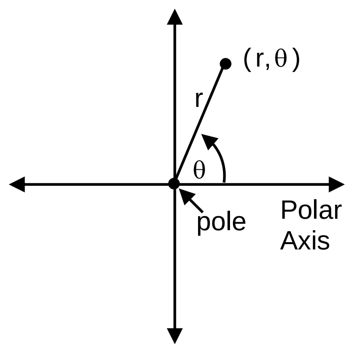
Section 6.1 – Polar Coordinates – Day 1

We have been using a system of rectangular coordinates to plot points. Now we will consider another system called polar coordinates.

We have been plotting graphs such as $f(\theta) = \cos\theta$ by letting θ be a distance along the horizontal axis. A more natural way to plot such graphs is for θ to be an angle in standard position. Then $f(\theta)$ is represented by a distance from the origin along the terminal side of the angle. A graph drawn this way is said to be in a polar coordinate system.

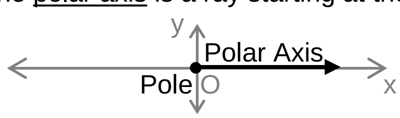


A point in Cartesian coordinates



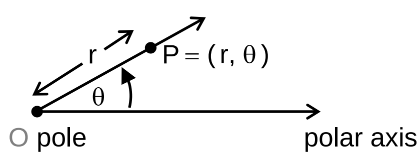
The same point in polar coordinates

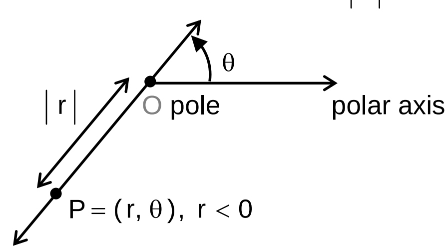
In a polar coordinate system, the pole is the point corresponding to the origin of a Cartesian coordinate system. The polar axis is a ray starting at the pole and going horizontally to the right.



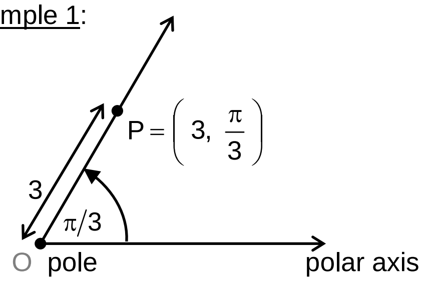
A point P in a polar coordinate system is represented by an ordered pair (r, θ). This is a reversal of the usual agreement that the independent variable comes first. The number r is the directed distance of the point from the pole, and θ is an angle (in degrees or radians) formed by the polar axis and a ray from the pole through the point. The ordered pair (r, θ) is called the polar coordinates of the point.

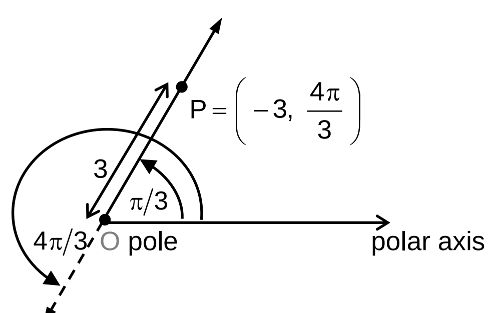
It is possible for the first entry r in (r, θ) to be negative. If r is negative, the location of the point is on the ray from the pole extending in the direction opposite the terminal side of θ at a distance $|r|$ from the pole.





Example 1:





These points, $\left(3, \frac{\pi}{3}\right)$ and $\left(-3, \frac{4\pi}{3}\right)$, are different names for the same point P.

This example shows a major difference between rectangular coordinates and polar coordinates. In rectangular coordinates, each point has exactly one pair of rectangular coordinates. In polar coordinates, a point can have infinitely many pairs of polar coordinates.

Section 6.1 – Polar Coordinates – Day 1 (continued)

So, a point with polar coordinates (r, θ) also can be represented by any of the following:

$(r, \theta + 360^\circ k)$ or $(-r, \theta + 180^\circ + 360^\circ k)$, where k is any integer.

Thus, for a positive value of r, the angle needs to be coterminal with θ, so add an integral multiple of 360°.
For a negative value of r, the angle must terminate half a revolution from θ, so add 180° and then an integral multiple of 360°.

The polar coordinates of the pole are $(0, \theta)$, where θ can be any angle.

Example 2: For the point with polar coordinates $\left(4, \frac{\pi}{6}\right)$, find other polar coordinates of this same point for which a) $r > 0$, $2\pi \le \theta < 4\pi$ and b) $r < 0$, $0 \le \theta < 2\pi$.

a) $r > 0$, $2\pi \le \theta < 4\pi$

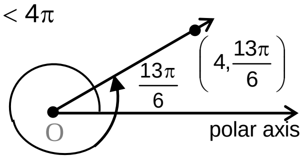
$r = 4$

$\theta = \frac{\pi}{6} + 2\pi$

$= \frac{\pi}{6} + \frac{12\pi}{6}$

$= \frac{13\pi}{6}$

$(r, \theta) = \left(4, \frac{13\pi}{6}\right)$



b) $r < 0$, $0 \le \theta < 2\pi$

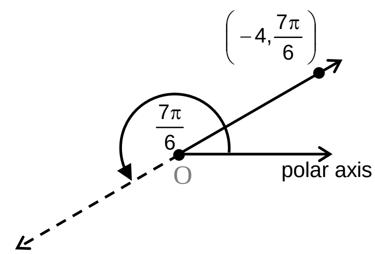
$r = -4$

$\theta = \frac{\pi}{6} + \pi$

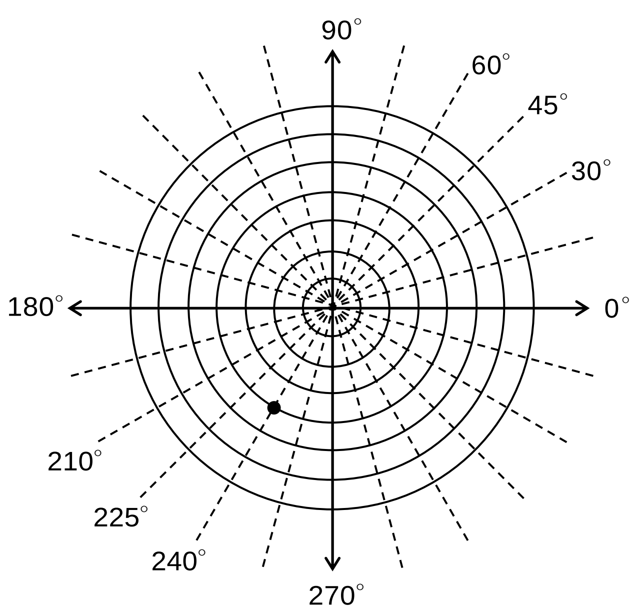
$= \frac{\pi}{6} + \frac{6\pi}{6}$

$= \frac{7\pi}{6}$

$(r, \theta) = \left(-4, \frac{7\pi}{6}\right)$



Example 3: Draw a graph of the given point in polar coordinates. Then give two other pairs of polar coordinates for the point, one with a positive value of r and the other with a negative value of r.
Point: $(-4, 60^\circ)$



$(-4, 60^\circ) = (-4, 60^\circ + 360^\circ)$

$= (-4, 420^\circ)$

$(-4, 60^\circ) = (4, 60^\circ + 180^\circ)$

$= (4, 240^\circ)$