Section 6.3 – Polar-Cartesian Transformations – Day 1

Theorem: Identifying Conics without Completing the Squares

The equation, $Ax^2 + Cy^2 + Dx + Ey + F = 0$, where either $A \neq 0$ or $C \neq 0$:

a) defines a Parabola if $AC = 0$

b) defines an Ellipse (or circle) if $AC > 0$ (Circle if $A = C$)

c) defines a Hyperbola if $AC < 0$

Example 1: Identify each equation without completing the squares.

a) $3x^2 + 6y^2 + 6x - 12y = 0$

$A = 3$, $C = 6$

$AC = 3(6)$

$= 18$

So, $AC > 0 \Rightarrow$ The equation is an Ellipse

b) $2x^2 - 3y^2 + 6y + 4 = 0$

$A = 2$, $C = -3$

$AC = 2(-3)$

$= -6$

So, $AC < 0 \Rightarrow$ The equation is a Hyperbola

c) $y^2 - 2x + 4 = 0$

$A = 0$, $C = 1$

$AC = 0(1)$

$= 0$

So, $AC = 0 \Rightarrow$ The equation is a Parabola

The standard equation of a parabola with its vertex at (h, k) is given below.

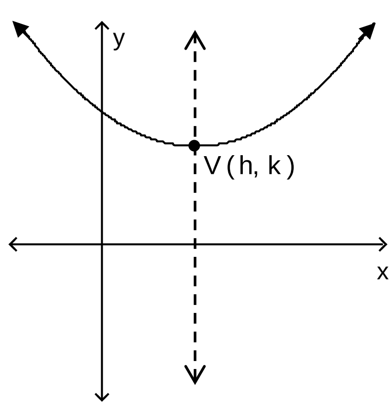
Vertical Parabola

$y - k = \frac{1}{4a}(x - h)^2$

$a > 0$: opens upward

$a < 0$: opens downward

vertex: $(h,\ k)$



Horizontal Parabola

$x - h = \frac{1}{4a}(y - k)^2$

$a > 0$: opens right

$a < 0$: opens left

vertex: $(h,\ k)$