Chapter 4 – Inverse Functions

When a function is defined by an equation in x and y, the graph of the function is the graph of the equation, which is the set of points (x, y) in the xy-plane that satisfies the equation. The graph of a function cannot contain two points with the same x-coordinate and different y-coordinates. The graph of a function must satisfy the vertical-line test.

Theorem: Vertical-line Test

A set of points in the xy-plane is the graph of a function if and only if every vertical line intersects the graph in at most one point.

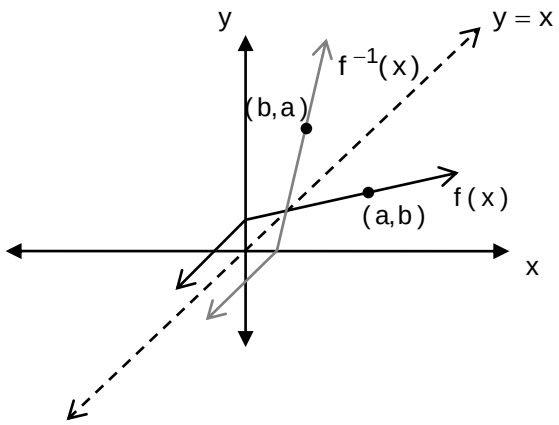
If the function f is the set of ordered pairs (x, y), then the inverse of f is the set of ordered pairs (y, x).
So, to find the inverse, interchange the elements in the domain with the elements in the range.

Example 1: Find the inverse of the function $f = \{(-4, -20), (3, -15), (0, 0), (2, 10)\}$.
The inverse of $f = \{(-20, -4), (-15, 3), (0, 0), (10, 2)\}$.

Theorem: The graph of a function $f(x)$ and the graph of its inverse $f^{-1}(x)$ are symmetric with respect to the line $y = x$.

So, if (a, b) is a point on the graph of f, then (b, a) is a point on the graph of f^{-1}.

Example 2:



To find the inverse of a function $f(x)$:

1) Replace $f(x)$ with y
2) Interchange the x and y variables
3) Solve the equation for y
4) Replace y with $f^{-1}(x)$

Example 3: Find the inverse of $f(x) = 3x + 5$.

Find f^{-1}:

$f(x) = 3x + 5$

$y = 3x + 5$

$x = 3y + 5$

$x - 5 = 3y$

$y = \frac{x - 5}{3}$

$f^{-1}(x) = \frac{x - 5}{3}$

Check: $f(f^{-1}(x)) = x$?

$f(f^{-1}(x)) = 3\left(\frac{x - 5}{3}\right) + 5$

$= (x - 5) + 5$

$= x$

$f^{-1}(f(x)) = x$?

$f^{-1}(f(x)) = \frac{(3x + 5) - 5}{3}$

$= \frac{3x}{3}$

$= x$

Chapter 4 – Inverse Functions Worksheet

Graph the given functions and determine if they are inverses of each other.

(Hint: Are they symmetric with respect to the line $y = x$? If (x, y) is on f(x), is (y, x) on $f^{-1}(x)$?)

1) $f(x) = \frac{x+8}{3}$, $g(x) = 3x - 8$

2) $f(x) = \frac{-1}{2}x + 3$, $g(x) = -2x + 6$

3) $f(x) = \frac{x-1}{4}$, $g(x) = 4x - 1$

4) $f(x) = \frac{-1}{3}x + 2$, $g(x) = -3x - 3$

Find the inverse function of the given function. Check your answers.

5) $f(x) = 8x - 1$

6) $f(x) = x + \frac{3}{5}$

7) $f(x) = \frac{2x-3}{4}$

8) $f(x) = \frac{x}{3} + 7$

Copy the given graph. Using the highlighted points, graph the inverse of the given function on the same graph. Label the function and its inverse.

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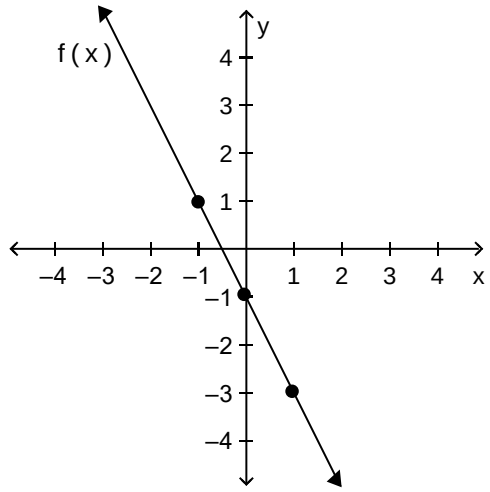

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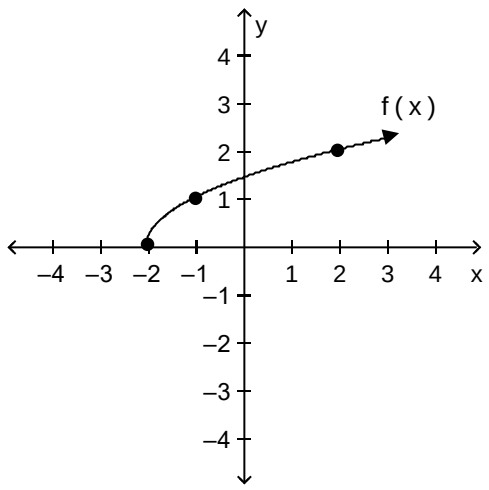

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