

Exercise 21.2b

Half-Lives – Answers

Name: _____

Date: _____ Per: _____

A nuclear half-life represents the amount of time required for one-half of a radioactive isotope (radionuclide) to decay. The decay that occurs in each half-life results in a characteristic pattern representing the remaining amount – 1/2, 1/4, 1/8, 1/16, 1/32, 1/64, etc. Amounts may be found using a table or using the formula:

$$\text{amount remaining} = \text{starting amount} \cdot (1/2)^n \text{ or } A_R = A_I(1/2)^n$$

where n = the number of half-lives

Sample:

If you start with 200 g of element Y (half-life 150 years), how many grams will be present 600 years from now?

$$A_R = 200\text{g} \cdot (1/2)^4 = 12.5\text{g}$$

($n = 4$ because 600 years represents 4 150 year half-lives.)

or

| Half-Lives | Time (years) | Amount Remaining |
|------------|--------------------|------------------|
| 0 | Now | 200 g |
| 1 | 150 years from now | 100 g |
| 2 | 300 years from now | 50 g |
| 3 | 450 years from now | 25 g |
| 4 | 600 years from now | 12.5 g |

DIRECTIONS: Solve the following in the space provided:

1. Solve the half-life formula for n .

$$\text{Base 10: } A_R = A_I(1/2)^n \Rightarrow A_R/A_I = (1/2)^n \Rightarrow \log(A_R/A_I) = \log(1/2)^n \Rightarrow \boxed{n = \log(A_R/A_I)/\log(1/2)}$$

$$\text{Base } 1/2: A_R = A_I(1/2)^n \Rightarrow A_R/A_I = (1/2)^n \Rightarrow \boxed{\log_{1/2}(A_R/A_I) = n}$$

2. Solve the half-life formula for “starting amount”.

$$A_R = A_I(1/2)^n \Rightarrow \boxed{A_I = A_R/(1/2)^n}$$

3. If you currently have 20. g of element X (half-life 300 years), how many grams would have been present 1500 years ago?

$$1500 \text{ years}/300 \text{ years} = 5 \text{ half-lives}$$

$$A_R = A_I(1/2)^n \Rightarrow A_I = A_R/(1/2)^n \Rightarrow A_I = 20.\text{g}/(1/2)^5 \Rightarrow A_I = 20.\text{g}/(1/32) \Rightarrow A_I = \boxed{640\text{g}} \text{ or}$$

| | |
|----------|------------------------------------|
| -1500 yr | 640 g |
| -1200 yr | 320 g |
| -900 yr | 160 g |
| -600 yr | 80. g |
| -300 yr | 40. g |
| Now | <input type="text" value="20. g"/> |

4. The half-life of plutonium-239 is 24300 years. If a nuclear bomb released 8 kg of this isotope, how many years would pass before the amount is reduced to 1 kg?

$$n = \log_{1/2}(A_R/A_I) \Rightarrow n = \log_{1/2}(1 \text{ kg}/8 \text{ kg}) \Rightarrow n = 3 \Rightarrow 3 \times 24300 \text{ yr} = \boxed{72900 \text{ yr}} \text{ or}$$

| | |
|-------|------|
| Start | 8 kg |
| 1 HL | 4 kg |
| 2 HL | 2 kg |
| 3 HL | 1 kg |

$$3 \times 24300 \text{ yr} = \boxed{72900 \text{ yr}}$$

5. The half-life of radon-222 is 3.8 days. How much of a 100. gram sample is left after 15.2 days?

$$15.2 \text{ days}/3.8 \text{ days} = 4 \text{ half-lives} \Rightarrow A_R = A_I(1/2)^n \Rightarrow A_R = 100.\text{g}/(1/2)^4 \Rightarrow A_R = \boxed{6.25 \text{ g}} \text{ or}$$

| | |
|---------|-------------------------------------|
| Now | 100. g |
| +3.8 d | 50.0 g |
| +7.6 d | 25.0 g |
| +11.4 d | 12.5 g |
| +15.2 d | <input type="text" value="6.25 g"/> |

6. Carbon-14 has a half-life of 5730 years. If a sample contained 70.0 mg originally, how much is left after 17190 years?

$$17190 \text{ yr}/5730 \text{ yr} = 3 \text{ half-lives} \Rightarrow A_R = A_I(1/2)^n \Rightarrow A_R = 70.0\text{mg}/(1/2)^3 \Rightarrow A_R = \boxed{8.75 \text{ mg}} \text{ or}$$

| | |
|-----------|--------------------------------------|
| Now | 70.0 mg |
| +5730 yr | 35.0 mg |
| +11460 yr | 17.5 mg |
| +17190 yr | <input type="text" value="8.75 mg"/> |

7. The half-life of cobalt-60 is 5.26 years. If 50.0 grams are left after 15.78 years, how many grams were in the original sample?

$$15.78 \text{ years}/5.26 \text{ years} = 3 \text{ half-lives}$$

$$A_R = A_I(1/2)^n \Rightarrow A_I = A_R/(1/2)^n \Rightarrow A_I = 50.0\text{g}/(1/2)^3 \Rightarrow A_I = 50.0\text{g}/(1/8) \Rightarrow A_I = \boxed{400\text{g}} \text{ or}$$

| | |
|-----------|-------------------------------------|
| -15.78 yr | <input type="text" value="400. g"/> |
| -10.52 yr | 200. g |
| -5.26 yr | 100. g |
| Now | 50.0 g |

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8. If you have 200. g of radioactive element Z at 12:00noon and have only 12.5g left at 8:00pm that same day, what is the half-life of this element?

$$n = \log_{1/2}(A_R/A_I) \Rightarrow n = \log_{1/2}(200. \text{ g}/12.5 \text{ g}) \Rightarrow n = 4 \Rightarrow 8:00\text{pm}-12:00\text{pm}/4 \text{ half-lives} = \boxed{2 \text{ hrs/half-life}}$$

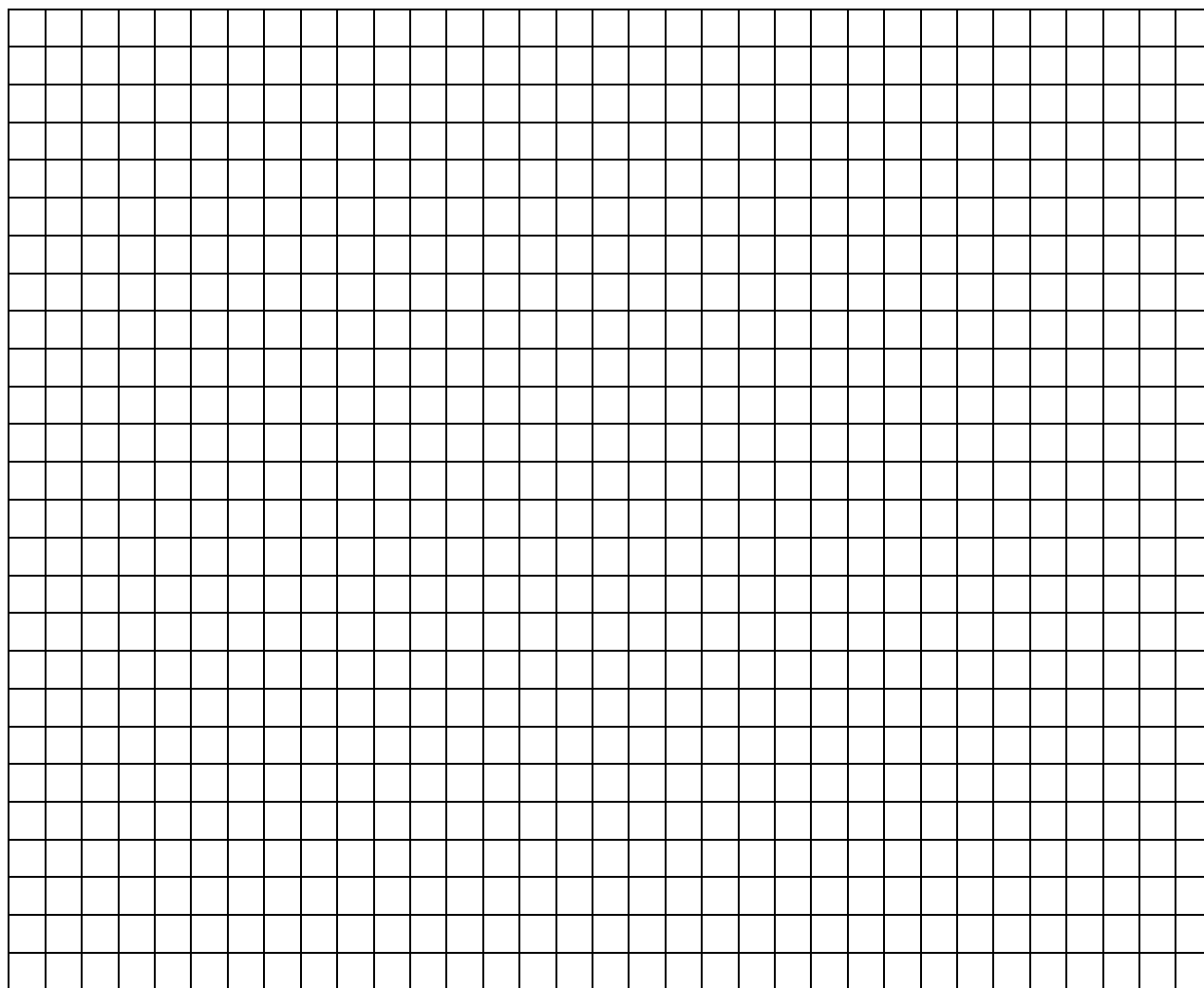
or

| | |
|-------|--------|
| Start | 200. g |
| +1 HL | 100. g |
| +2 HL | 50.0 g |
| +3 HL | 25.0 g |
| +4 HL | 12.5 g |

It takes 4 half-lives for the sample to decay from 200. g to 12.5 g. Since the four half-lives occur over 8 hours, the half-life is 2 hours.

9. Graph the following data on the graph, then use the graph to determine the half-life of this isotope.

| <i>Time (years)</i> | <i>0</i> | <i>1</i> | <i>2</i> | <i>3</i> | <i>4</i> | <i>5</i> | <i>6</i> | <i>7</i> | <i>8</i> | <i>9</i> | <i>10</i> |
|-------------------------------|------------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|----------|-----------|
| <i>Mass Remaining (grams)</i> | 100 | 75 | 56 | 42 | 32 | 24 | 18 | 13 | 10 | 8 | 6 |



Half-life = about 2.4 years

DIRECTIONS: Answer the following in the space provided.

10. How are fission & fusion alike: _____

11. How are fission & fusion different: _____

12. What does $E=mc^2$ really mean to us? _____