Appendix A.6– Solving Equations – Day 2

Solve a Quadratic Equation by Factoring

A quadratic equation is an equation equivalent to one of the form $ax^2 + bx + c = 0$, where a, b, and c are real numbers and $a \neq 0$.

A quadratic equation written in the form $ax^2 + bx + c = 0$ is said to be in **standard form**.

Sometimes, a quadratic equation is called a **second-degree equation** because, when it is in standard form, the left side is a polynomial of degree 2.

When a quadratic equation is written in standard form, it may be possible to factor the expression on the left side as the product of two first-degree polynomials. Then, by using the Zero-Product Property and setting each factor equal to 0, the resulting linear equations can be solved to obtain the solutions of the quadratic equation.

Example 6: Solve the equation $x^2 = 15 - 2x$.

$$x^2 + 2x - 15 = 0$$
$$(x+5)(x-3) = 0$$
$$\Rightarrow \quad x + 5 = 0 \text{ or } x - 3 = 0$$
$$x = -5 \text{ or } x = 3$$

The solution set is $\{-5,\ 3\}$.

When the left side factors into two linear equations with the same solution, the quadratic equation is said to have a **repeated solution**. This solution is also called a **root of multiplicity 2**, or a **double root**.

Example 7: Solve the equation $x^2 - 14x + 49 = 0$.

$$x^2 - 14x + 49 = 0$$
$$(x-7)(x-7) = 0$$
$$\Rightarrow \quad x - 7 = 0$$
$$x = 7$$

The solution set is $\{\ 7\}$. 7 is a repeated solution or a double root.

You use the **Square Root Method** to solve a quadratic equation if $x^2 = p$ where $p \geq 0$. Then the solution is $x = \sqrt{p}$ or $x = -\sqrt{p}$. We usually abbreviate these solutions as $x = \pm\sqrt{p}$, which is read as "x equals plus or minus the square root of p."

Example 8: Solve the equation $x^2 = 36$.

$$x^2 = 36$$
$$x = \pm\sqrt{36}$$
$$x = \pm 6$$

The solution set is $\{\ -6,\ 6\}$.

Solve a Quadratic Equation by Completing the Square

The idea behind the **Completing the Square Method** is to adjust the left side of a quadratic equation, $ax^2 + bx + c$, so that it becomes a perfect square, i.e., the square of a first-degree polynomial. We adjust the left side by adding the appropriate number to the left side to create a perfect square.

Appendix A.6– Solving Equations – Day 2 (continued)

Procedure for Completing the Square:

To complete the square for any quadratic expression of the form x^2+bx:

1) Find one half of b, the coefficient of x.
2) Square the result in step 1.
3) Add the result of step 2 to x^2+bx

So, $x^2+bx+\left(\frac{b}{2}\right)^2=\left(x+\frac{b}{2}\right)^2$.

Example 9: Solve the equation $x^2-8x+15=0$ by completing the square.

$x^2-8x+15=0 \Rightarrow$ the x^2 term has a coefficient of 1 and $b=-8$

So, $\frac{b}{2}=\frac{-8}{2}=-4$ and $\left(\frac{b}{2}\right)^2=\left(\frac{-8}{2}\right)^2=(-4)^2=16$

Isolate the x^2+bx portion of the equation:

$x^2-8x=-15$

Add 16 to both sides of the equation:

$x^2-8x+16=-15+16$

$(x-4)^2=1$

$x-4=\pm\sqrt{1}$

$x-4=\pm 1$

$x=4\pm 1$

$x=5$ or $x=3$

The solution set is $\{3,\ 5\}$.

Quadratic equations that cannot be easily factored can be solved by using the **quadratic formula**.

Quadratic Formula Theorem:

Given the quadratic equation $ax^2+bx+c=0$, where $a\neq 0$, then

If $b^2-4ac<0$, the equation has no real solution.

If $b^2-4ac\geq 0$, the real solution(s) of the equation is(are) given by the quadratic formula:

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}.$$

Recall that b^2-4ac is called the **discriminant**. It tells you how many real solutions a quadratic equation has:

1) If $b^2-4ac>0$, the equation has two unequal real solutions.
2) If $b^2-4ac=0$, the equation has one repeated real solution, a double root.
3) If $b^2-4ac<0$, the equation has no real solution.

Example 10: Solve the equation $2x^2-2x+3=0$.

$2x^2-2x+3=0 \Rightarrow a=2,\ b=-2,\ c=3$

So, $b^2-4ac=(-2)^2-4(2)(3)$
$=4-24$
$=-20$

Since the discriminant $b^2-4ac<0$, the equation has no real solution.

Appendix A.6– Solving Equations – Day 2 (continued)

Example 11: Solve the equation $8x^2+6x-1=0$ by using the quadratic formula.

$$8x^2+6x-1=0 \Rightarrow a=8,\ b=6,\ c=-1$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-6\pm\sqrt{6^2-4(8)(-1)}}{2(8)}$$

$$=\frac{-6\pm\sqrt{36+32}}{16}$$

$$=\frac{-6\pm\sqrt{68}}{16}$$

$$=\frac{-6\pm\sqrt{4}\sqrt{17}}{16}$$

$$=\frac{-6\pm2\sqrt{17}}{16}$$

$$=\frac{\cancel{2}(-3\pm\sqrt{17})}{\cancel{2}(8)}$$

$$=\frac{-3\pm\sqrt{17}}{8}$$

The solution set is $\left\{\frac{-3+\sqrt{17}}{8},\ \frac{-3-\sqrt{17}}{8}\right\}$.

Procedure for Solving a Quadratic Equation:

To solve a quadratic equation, first put the equation in standard form: $ax^2+bx+c=0,\quad a\neq0$

Then:

1) Identity a, b, and c.
2) Evaluate the discriminant, b^2-4ac.
3) (a) If the discriminant is negative, the equation has no real solution
   (b) If the discriminant is zero, the equation has one real solution, a repeated root.
   (c) If the discriminant is positive, the equation has two distinct real solutions. If you can easily spot factors, use the factoring method to solve the equation. Otherwise, use the quadratic formula or the method of completing the square.

All material has been taken from *Precalculus,* by M. Sullivan, 10th Edition