Appendix A.1 – Algebra Essentials

A set is a collection of similar but distinct objects, where each object is called an **element** of the set.

There are two methods of denoting a set.

Method 1: Roster Method – use braces { } to enclose all elements in the set.
Example – The set of all even numbers between 1 and 9 = { 2, 4, 6, 8 }

Method 2: Set-Builder Notation – use a variable to help denote a set.
E = { x | x is an even number between 1 and 9 }.
Read as "E is the set of all x such that x is an even number between 1 and 9."

Subset – If every element of a set A is also an element of a set B, then we say that A is a subset of B.
Example: Let A = { 1, 2 } and B = { 1, 2, 3, 4 }. Then A is a subset of B.

An empty set, or null set, is a set that has no elements. It is denoted by the symbol Ø.

Sets of Numbers

Real Numbers – The set of numbers consisting of the positive numbers, the negative numbers, and zero.

Counting or Natural Numbers – { 1, 2, 3, 4, … }

Integers – { …, –3, –2, –1, 0, 1, 2, 3, … }

Rational Numbers – Any number that can be expressed as a quotient $\frac{a}{b}$ of two integers, where integer b cannot be 0.

Rational real numbers have decimal representations that either terminate or are nonterminating with repeating blocks of digits. Example: $\frac{1}{3} = 0.333...$

Irrational Numbers – Any number that cannot be expressed as a quotient $\frac{a}{b}$ of two integers, i.e., a real number that is not a rational number. Examples: $\sqrt{2}$ and π.

Irrational real numbers have decimal representations that neither repeat nor terminate.

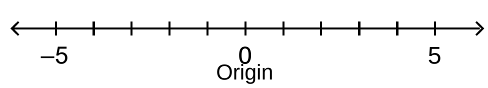
Two frequently used properties of real numbers are the Distributive and Zero-Product Properties. Suppose that *a*, *b*, and *c* are real numbers.

Distributive Property: $a(b+c) = ab + ac$

Zero-Product Property: If $ab = 0$, then either $a = 0$ or $b = 0$ or both equal 0.

The Zero-Product Property will be used to solve equations.

The Real Number Line



The real numbers can be represented by points on a line called the real number line. Every real number corresponds to a point on the line, and each point on the line has a unique real number associated with it. The real number associated with a point P is called the coordinate of P. The point on the line corresponding to the real number 0 is called the **origin**.

The real number line consists of three classes of real numbers:

1) The negative real numbers are the coordinates of points to the left of the origin.
2) The real number 0 is the coordinate of the origin.
3) The positive real numbers are the coordinates of points to the right of the origin.

Appendix A.1 – Algebra Essentials (continued)

Inequalities

The symbols $<$, $>$, $\leq$, and $\geq$ are called **inequality symbols**. An **inequality** is a statement in which two expressions are related by an inequality symbol. The expressions are referred to as the sides of the inequality. Statements of the form $a < b$ or $b > a$ are called **strict inequalities**. Statements of the form $a \leq b$ or $b \geq a$ are called **nonstrict inequalities**. Note that $a < b$ and $b > a$ mean the same thing.

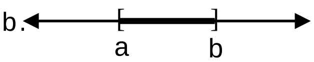
$a > 0$ is equivalent to a is positive.
$a < 0$ is equivalent to a is negative.

The inequality $a > 0$ is sometimes read as "a is positive." If $a \geq 0$, then either $a > 0$ or $a = 0$, and this is read as "a is nonnegative."

Intervals

Let a and b represent two real numbers with $a < b$.

A **closed interval**, denoted by $[a, b]$, consists of all real numbers x for which $a \leq x \leq b$.



An **open interval**, denoted by (a, b), consists of all real numbers x for which $a < x < b$.

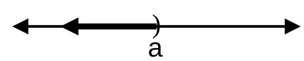
The **half-open**, or **half-closed, intervals** are $(a, b]$, consisting of all real numbers x for which $a < x \leq b$, and $[a, b)$, consisting of all real numbers x for which $a \leq x < b$.

a is called the left endpoint and b is called the right endpoint of the interval in each of the above definitions.

The symbol ∞ (infinity) is not a real number, but a notational device used to indicate unboundedness in the positive direction. The symbol $-\infty$ (negative infinity) also is not a real number, but a notational device used to indicate unboundedness in the negative direction.

Using ∞ and $-\infty$, we can define five other kinds of intervals:

$[a, \infty)$	Consists of all real numbers x for which $x \geq a$ $(a \leq x < \infty)$
(a, ∞)	Consists of all real numbers x for which $x > a$ $(a < x < \infty)$
$(-\infty, a]$	Consists of all real numbers x for which $x \leq a$ $(-\infty < x \leq a)$
$(-\infty, a)$	Consists of all real numbers x for which $x < a$ $(-\infty < x < a)$
$(-\infty, \infty)$	Consists of all real numbers x $(-\infty < x < \infty)$



Note that $-\infty$ and ∞ are never included as endpoints, since neither is a real number.

Absolute Value

The absolute value of a number A, $|A|$, is the distance on a number line from the point whose coordinate is A to the origin.

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The **absolute value** of a real number *a*, denoted by the symbol $|a|$, is defined by the rules

$|a| = a$ if $a \geq 0$ and $|a| = -a$ if $a < 0$.

Examples: $|7| = 7$ and $|-12| = 12$

Appendix A.1 – Algebra Essentials (continued)

If P and Q are two points on a real number line with coordinates *a* and *b*, respectively, the **distance between P and Q**, denoted by d(P,Q) is $|b-a|$.

Since $|b-a| = |a-b|$, it follows that d(P,Q) = d(Q,P).

Let P, Q, and R be points on the real number line with coordinates -4, 9, and -6, respectively.

Then
$$\begin{aligned} d(P,Q) &= |9-(-4)| \\ &= |9+4| \\ &= |13| \\ &= 13 \end{aligned} \qquad \begin{aligned} d(Q,R) &= |-6-9| \\ &= |-15| \\ &= 15 \end{aligned}$$

Evaluate Algebraic Expressions

In algebra we use letters to represent numbers. A **variable** is a letter used to represent any number from a given set of numbers. A **constant** is either a fixed number, such as 4, or a letter that represents a fixed (possibly unspecified) number.

Constants and variables are combined using the operations of addition, subtraction, multiplication, and division to form algebraic expressions. Examples of algebraic expressions include $x+3$, $\frac{5}{1-t}$, and $7x-2y$. To evaluate an algebraic expression, substitute a numerical value for each variable.

The **domain** of a variable is the set of values that the variable in an expression may assume. Depending on the expression or formula, variables may be allowed to take on values from only a certain set of numbers. For example, in the expression $\frac{1}{6-x}$, the variable x cannot take on the value 6, since division by zero is not defined.

In describing the domain of a variable, you may use either set notation or words, whichever is more convenient. For the example above, the domain is $\{x \mid x \neq 6\}$ or the set of all real numbers other than 6.

Exponents

Recall: If *a* is a real number and *n* is a positive integer, then $a^n = \underbrace{a(a)(a)\ldots(a)}_{\text{n times}}$.

a^n: *a* is called the base and *n* is called the exponent, or power. We read a^n as "*a* raised to the power *n*" or as "*a* to the *n*th power." In working with exponents, the operation of *raising to a power* is performed before any other operation. For example,
$$\begin{aligned} 4\cdot 3^2 &= 4\cdot 9 \\ &= 36 \end{aligned}$$

Be careful with minus signs and exponents.
$$\begin{aligned} -2^4 &= -1(2^4) \\ &= -16 \end{aligned} \quad \text{whereas} \quad \begin{aligned} (-2)^4 &= (-2)(-2)(-2)(-2) \\ &= (4)(4) \\ &= 16 \end{aligned}$$

If $a \neq 0$, then $a^0 = 1$.

If $a \neq 0$ and if *n* is a positive integer, then $a^{-n} = \frac{1}{a^n}$.

Whenever you encounter a negative exponent, think "reciprocal."

Appendix A.1 – Algebra Essentials (continued)

Laws of Exponents:

$a^m\left(a^n\right) = a^{m+n}$ $\quad (a^m)^n = a^{m(n)}$ $\quad (ab)^n = a^n\left(b^n\right)$ $\quad$ a and b are real numbers, m and n are integers

$\frac{a^m}{a^n} = a^{m-n} = \frac{1}{a^{n-m}}$, if $a \neq 0$ $\quad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$, if $b \neq 0$

Write expressions so that all exponents are positive.

Examples:

$$\frac{x^6y^{-2}}{x^2y} = x^6x^{-2}y^{-2}y^{-1} = x^4y^{-3} = \frac{x^4}{y^3}$$

$$\frac{xy}{x^{-1}-y^{-1}} = \frac{xy}{\frac{1}{x}-\frac{1}{y}} = \frac{xy}{\frac{y-x}{xy}} = xy\left(\frac{xy}{y-x}\right) = \frac{x^2y^2}{y-x}$$

Evaluate Square Roots

A real number is squared when it is raised to the power 2. The inverse of squaring is finding a square root. For example, since $6^2 = 36$ and $(-6)^2 = 36$, the numbers 6 and -6 are square roots of 36.
The symbol $\sqrt{}$, called a **radical sign**, is used to denote the **principal**, or nonnegative, square root. For example, $\sqrt{36} = 6$.

If *a* is a nonnegative real number, the nonnegative number *b* such that $b^2 = a$ is the **principal square root** of *a*, and is denoted by $b = \sqrt{a}$.

The following comments are noteworthy:

1. Negative numbers do not have square roots (in the real number system), because the square root of any real number is *nonnegative*. For example, $\sqrt{-4}$ is not a real number, because there is no real number whose square is -4.
2. The principal square root of 0 is 0, since $0^2 = 0$. That is, $\sqrt{0} = 0$.
3. The principal square root of a positive number is positive.
4. If $c \geq 0$, then $\left(\sqrt{c}\right)^2 = c$. For example, $\left(\sqrt{2}\right)^2 = 2$ and $\left(\sqrt{3}\right)^2 = 3$.

Examples:

$\sqrt{64} = 8$ $\quad \sqrt{\frac{1}{16}} = \frac{1}{4}$ $\quad \sqrt{(1.4)^2} = 1.4$

Consider the expression $\sqrt{a^2}$. Since $a^2 \geq 0$, the principal square root of a^2 is defined whether $a > 0$ or $a < 0$. However, since the principal square root is nonnegative, you need an absolute value to ensure the nonnegative result. That is,

$\sqrt{a^2} = |a|$ for *a* any real number

Appendix A.1 – Algebra Essentials (continued)

Calculators

Calculators are incapable of displaying decimals that contain a large number of digits. For example, some calculators are capable of displaying only eight digits. When a number requires more than eight digits, the calculator either truncates or rounds. To see how your calculator handles decimals, divide 2 by 3. How many digits do you see? Is the last digit a 6 or a 7? If it is a 6, your calculator truncates; if it is a 7, your calculator rounds.

There are different kinds of calculators. An **arithmetic** calculator can only add, subtract, multiply, and divide numbers; therefore, this type is not adequate for this course. **Scientific** calculators have all the capabilities of arithmetic calculators and also contain **function keys** labeled log, sin, cos, tan, x^y, inv, and so on. **Graphing** calculators have all the capabilities of scientific calculators and contain a screen on which graphs can be displayed. This textbook uses a small picture of a calculator whenever a graphing calculator needs to be used.

Use a Calculator to Evaluate Exponents

Your calculator has either a caret key, ^, or an x^y key, that is used for computations involving exponents.

Example: Evaluate $(2.3)^5$.

Your calculator gives you $(2.3)^5 = 64.36343$.

All material has been taken from *Precalculus,* by M. Sullivan, 10th Edition