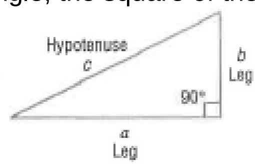
Appendix A.2 – Geometry Essentials, Day 1

Use the Pythagorean Theorem and Its Converse

The Pythagorean Theorem is a statement about *right triangles*. A **right triangle** is a triangle that contains a **right angle** – that is, an angle of 90°. The side of the triangle opposite the 90° angle is called the **hypotenuse**; the remaining two sides are called **legs**.

Pythagorean Theorem: In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.



Example 1: In a right triangle, one leg is of length 6 and the other leg is of length 8. What is the length of the hypotenuse?

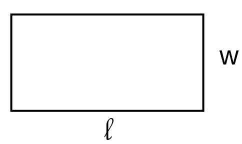
$$\text{hyp}^2 = \text{leg}_1^2 + \text{leg}_2^2$$
$$\text{hyp}^2 = 6^2 + 8^2$$
$$= 36 + 64$$
$$= 100$$
$$\text{hyp} = \sqrt{100}$$
$$= 10$$

Converse of the Pythagorean Theorem: In a triangle, if the square of the length of one side equals the sum of the squares of the lengths of the other two sides, then the triangle is a right triangle. The 90° angle is opposite the longest side.

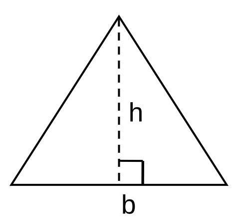
Know Geometry Formulas

Certain formulas from geometry are useful in solving algebra problems.

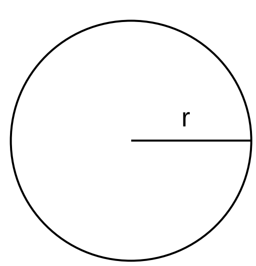
For a rectangle of length ℓ and width w,
$\text{Area} = \ell w$ $\quad$ $\text{Perimeter} = 2\ell + 2w$



For a triangle with base b and altitude h,
$\text{Area} = \frac{1}{2}bh$



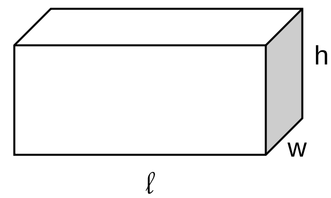
For a circle of radius r (diameter $d = 2r$),
$\text{Area} = \pi r^2$ $\quad$ $\text{Circumference} = 2\pi r$
$= \pi d$



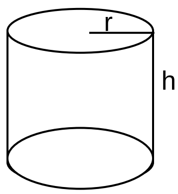
Appendix A.2 – Geometry Essentials (continued)

For a closed rectangular box of length ℓ, width w, and height h,

$\text{Volume} = \ell w h \qquad \text{Surface Area} = 2\ell w + 2wh + 2\ell h$

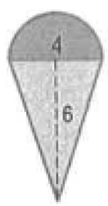


For a closed right circular cylinder of height *h* and radius *r*,



$\text{Volume} = \pi r^2 h, \quad \text{Surface area} = 2\pi r^2 + 2\pi r h$

Example 2: An ornament is in the shape of a semicircle on top of a triangle. How many square centimeters (cm^2) of copper is required to make the ornament if the height of the triangle is 6 cm and the base is 4 cm?



The amount of copper required equals the shaded area. This area is the sum of the areas of the triangle and the semicircle. The triangle has height $h = 6$ and base $b = 4$. The semicircle has diameter $d = 4$, so its radius is $r = 2$.

$$\begin{aligned} \text{Area} &= \text{Area of Triangle} + \text{Area of Semicircle} \\ &= \frac{1}{2}bh + \frac{1}{2}\pi r^2 \\ &= \frac{1}{2}(4)(6) + \frac{1}{2}\pi(2)^2 \\ &= \frac{24}{2} + \frac{4\pi}{2} \\ &= 12 + 2\pi \\ &\approx 18.283 \\ &\approx 18.28 \text{ cm}^2 \end{aligned}$$

About 18.28 cm^2 of copper is required.