Appendix A.6 – Solving Equations – Day 1

An **equation in one variable** is a statement in which two expressions, at least one containing the variable, are equal. The expressions are called the **sides** of the equation. Since an equation is a statement, it may or may not be true, depending on the value of the variable. The admissible values of the variable are those in the domain of the variable. Values of the variable that result in a true statement are called **solutions**, or **roots**, of the equation. To **solve an equation** means to find all the solutions of the equation.

We often write solutions of an equation in set notation. This set is called the **solution set** of the equation. For example, the solution set of the equation $x^2 - 16 = 0$ is $\{-4, 4\}$. Some equations have no real solution.

An equation that is satisfied for every value of the variable for which both sides are defined is called an **identity**. For example, the equation $2x + 3 = x + 2 + x + 1$ is an identity, since the statement is true for any real number x.

One method for solving an equation is to replace the original equation by a succession of **equivalent equations**, equations having the same solution set, until an equation with an obvious solution is obtained.

Procedures that Result in Equivalent Equations

1) Interchange the two sides of the equation:
   Replace $4 = x$ by $x = 4$

2) Simplify the sides of the equation by combining like terms, eliminating parentheses, and so on:
   Replace $(x+3)+7 = 4x + (x+2)$
   by $x + 10 = 5x + 2$

3) Add or subtract the same expression on both sides of the equation:
   Replace $2x - 7 = 3$
   by $(2x-7)+7 = 3+7$

4) Multiply or divide both sides of the equation by the same nonzero expression
   Replace $\frac{2x}{x-4} = \frac{6}{x-4}$, $\text{Dom} = \{x \mid x \neq 4\}$
   by $\left(\frac{2x}{x-4}\right)(x-4) = \left(\frac{6}{x-4}\right)(x-4)$

5) If one side of the equation is 0 and the other side can be factored, then use the Zero-Product Property and set each factor equal to 0:
   Replace $x(x-2) = 0$
   by $x = 0$ or $x - 2 = 0$

Solving an Equation

Example 1: Solve the equation $4x - 9 = 3$.

$4x - 9 = 3$

$4x - 9 + 9 = 3 + 9$

$4x = 12$

$\frac{4x}{4} = \frac{12}{4}$

$x = 3$

Example 2: Solve $\frac{4x}{x-1} + 2 = \frac{4}{x-1}$.

Domain: $\{x \mid x \neq 1\}$

$(x-1)\left(\frac{4x}{x-1} + 2\right) = \cancel{(x-1)}\left(\frac{4}{\cancel{x-1}}\right)$

$\cancel{(x-1)}\left(\frac{4x}{\cancel{x-1}}\right) + 2(x-1) = 4$

$4x + 2x - 2 = 4$

$6x - 2 = 4$

$6x - 2 + 2 = 4 + 2$

$6x = 6$

$\frac{6x}{6} = \frac{6}{6}$

$x = 1$ But $x = 1$ is not in the domain of the variable. So, there is no real solution.

Appendix A.6 – Solving Equations – Day 1 (continued)

Solving Equations by Factoring

Example 3: Solve the equation $x^3 = 36x$.

$$x^3 = 36x$$
$$x^3 - 36x = 0$$
$$x(x^2 - 36) = 0$$
$$x(x+6)(x-6) = 0$$
$$\Rightarrow \; x = 0 \;\text{ or }\; x+6=0 \;\text{ or }\; x-6=0$$
$$x = 0 \;\text{ or }\; x = -6 \;\text{ or }\; x = 6$$

The solution set is $\{-6,\ 0,\ 6\}$.

Example 4: Solve the equation $x^3 - x^2 - 9x + 9 = 0$.

$$x^3 - x^2 - 9x + 9 = 0$$
$$x^2(x-1) - 9(x-1) = 0$$
$$(x^2 - 9)(x-1) = 0$$
$$(x+3)(x-3)(x-1) = 0$$
$$\Rightarrow \; x+3=0 \;\text{ or }\; x-3=0 \;\text{ or }\; x-1=0$$
$$x = -3 \;\text{ or }\; x = 3 \;\text{ or }\; x = 1$$

The solution set is $\{-3,\ 0,\ 3\}$.

Solve Equations Involving Absolute Value

Example 5: Solve $|2x+3| = 15$.

Two possibilities: $2x+3 = 15$ or $-(2x+3) = 15$

$$2x+3-3 = 15-3 \qquad\qquad 2x+3 = -15$$
$$2x = 12 \qquad\qquad 2x+3-3 = -15-3$$
$$\frac{2x}{2} = \frac{12}{2} \qquad\qquad 2x = -18$$
$$x = 6 \qquad\qquad \frac{2x}{2} = \frac{-18}{2}$$
$$x = -9$$

The solution set is $\{-9,\ 6\}$.

All material has been taken from *Precalculus,* by M. Sullivan, 10th Edition