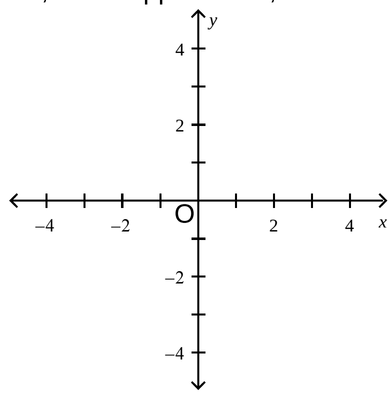
Section 1.1 – The Distance and Midpoint Formulas

Rectangular Coordinates

We locate a point on the real number line by assigning it a single real number, called the *coordinate of the point*. For work in a two-dimensional plane, we locate points by using two numbers.

Begin with two real number lines located in the same plane: one horizontal and the other vertical. The horizontal line is called the **x-axis**, the vertical line the **y-axis**, and the point of intersection the **Origin O**. Assign coordinates to every point on these number lies using a convenient scale. In mathematics, we usually use the same scale on each axis, but in applications, different scales appropriate to the application may be used.



The origin O has a value of 0 on both the x-axis and the y-axis. Points on the x-axis to the right of O are associated with positive real numbers, and those to the left of O are associated with negative real numbers. Points on the y-axis above O are associated with positive real numbers, and those below O are associated with negative real numbers. The x-axis and y-axis are labeled as x and y, respectively, and an arrow at the end of each axis is used to denote the positive direction.

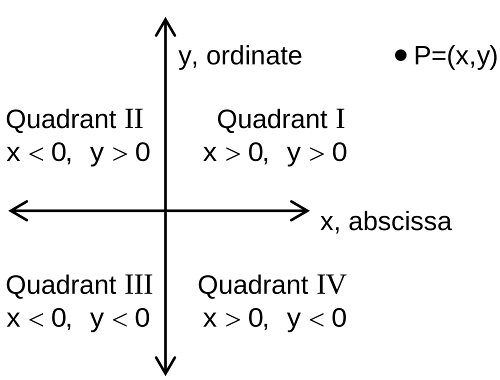
The coordinate system described here is called a **rectangular** or **Cartesian coordinate system**. The plane formed by the x-axis and y-axis is sometimes called the **xy-plane**, and the x-axis and y-axis are referred to as the **coordinate axes**.

Any point P in the xy-plane can be located by using an **ordered pair** (x, y) of real numbers. Let *x* denote the signed distance of P from the y-axis (*signed* means that if P is to the right of the y-axis, then x > 0, and if P is to the left of the y-axis, then x < 0); and let *y* denote the signed distance of P from the x-axis. The ordered pair (x, y), also called the **coordinates** of P, gives us enough information to locate the point P in the plane.

The origin has coordinates (0, 0). Any point on the x-axis has coordinates of the form (x, 0), and any point on the y-axis has coordinates of the form (0, y).

If (x, y) are the coordinates of a point P, then x is called the **x-coordinate**, or **abscissa**, of P and y is the **y-coordinate**, or **ordinate**, of P. We identify the point P by its coordinates (x, y) by writing P = (x, y). Usually, we will simply say "the point (x, y)" rather than "the point whose coordinates are (x, y)."

The coordinate axes divide the xy-plane into four sections, called quadrants. In quadrant I, both the x-coordinate and the y-coordinate of all points are positive; in quadrant II, x is negative and y is positive; in quadrant III, both x and y are negative; and in quadrant IV, x is positive and y is negative. Points on the coordinate axes belong to no quadrant.



Section 1.1 – The Distance and Midpoint Formulas (continued)

Use the Distance Formula

If the same units of measurement (such as inches, centimeters, and so on) are used for both the x-axis and y-axis, then all distances in the xy-plane can be measured using this unit of measurement.

The distance formula provides a method for computing the distance between two points in the xy-plane.

Distance Formula – The distance between two points $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$, denoted by $d(P_1, P_2)$, is

$$d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

So, to compute the distance between two points, find the difference of the x-coordinates, square it, and add this to the square of the difference of the y-coordinates. The square root of this sum is the distance.

The distance between two points is never negative. The distance between two points is 0 only when the points are identical. Also, $d(P_1, P_2) = d(P_2, P_1)$, i.e., the order of the points is not important.

Example 1: Find the distance between points P_1 and P_2. $P_1 = (4, -3)$ and $P_2 = (6, 4)$.

$$P_1 = (4, -3) \qquad P_2 = (6, 4)$$
$$= (x_1, y_1) \qquad = (x_2, y_2)$$
$$\begin{aligned} d(P_1, P_2) &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(6-4)^2 + (4-(-3))^2} \\ &= \sqrt{(2)^2 + (7)^2} \\ &= \sqrt{4+49} \\ &= \sqrt{53} \end{aligned}$$

The midpoint formula is used to calculate the midpoint of a line segment. Let $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$ be the endpoints of a line segment, and let $M = (x, y)$ be the point on the line segment that is the same distance from P_1 as it is from P_2.

Midpoint Formula – The midpoint M = (x, y) of the line segment from $P_1 = (x_1, y_1)$ to $P_2 = (x_2, y_2)$ is

$$\text{Midpoint M: } (x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right).$$

So, to find the midpoint of a line segment, average the x-coordinates of the endpoints and average the y-coordinates of the endpoints.

Example 2: Find the midpoint of the line segment from $P_1 = (-3, 2)$ to $P_2 = (6, 0)$.

$$P_1 = (-3, 2) \qquad P_2 = (6, 0)$$
$$= (x_1, y_1) \qquad = (x_2, y_2)$$
$$\begin{aligned} \text{Midpoint } M &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left(\frac{-3+6}{2}, \frac{2+0}{2} \right) \\ &= \left(\frac{3}{2}, \frac{2}{2} \right) \\ &= \left(\frac{3}{2}, 1 \right) \end{aligned}$$

All material has been taken from *Precalculus,* by M. Sullivan, 10th Edition