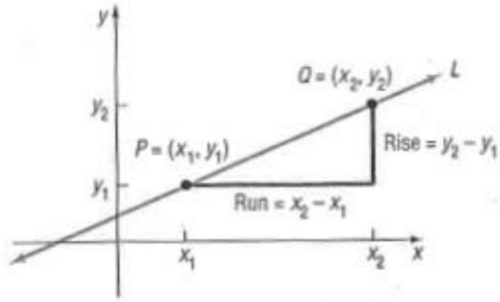
Section 1.3 – Lines

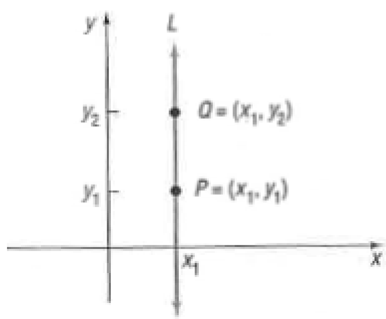
A linear equation is an equation that contains two variables and whose graph is a line.

A numerical measure of the steepness of a line is called the **slope**. The slope m of a line containing points (x_1, y_1) and (x_2, y_2) is defined to be $m = \frac{y_2 - y_1}{x_2 - x_1}$, where $x_1 \neq x_2$. Slope is also known as $\frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{\text{change in } y}{\text{change in } x}$. Slope is the **average rate of change** of y with respect to x.



(a) Slope of L is $m = \frac{y_2 - y_1}{x_2 - x_1}$, $x_1 \neq x_2$

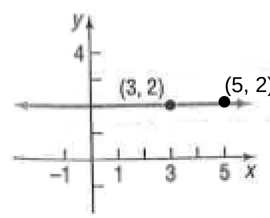
If $x_1 = x_2$, then you have a vertical line. A **vertical line** is given by an equation of the form x = a, where a is the x-intercept. The slope of a vertical line is undefined.



(b) Slope is undefined; L is vertical

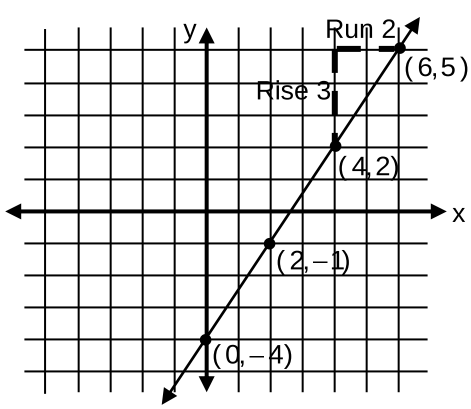
If $y_1 = y_2$, then you have a **horizontal line** given by an equation of the form y = b, where b is the y-intercept. The slope of a horizontal line is zero.

Graph $y = 2$.



Any two distinct points on a line, say P and Q, may be used to compute the slope of the line. The slope may be computed from P to Q or from Q to P. Lines with positive slope slant upwards from lower left to upper right. Lines with negative slope slant downwards from upper left to lower right.

Example 1: Draw a graph of the line that passes through the point (4, 2) and has a slope of $\frac{3}{2}$.



Section 1.3 – Lines (continued)

There are several forms of the equation of a line. Let's review them.

Point-Slope Form of an Equation of a Line

An equation of a nonvertical line of slope m that passes through the point (x_1, y_1) is $y - y_1 = m(x - x_1)$.

Example 2: Write the equation of the line with slope 3 that passes through the point $(-2, 4)$.

$$y - y_1 = m(x - x_1)$$
$$y - 4 = 3(x - (-2))$$
$$y - 4 = 3(x + 2)$$
$$y - 4 = 3x + 6$$
$$y = 3x + 10$$

Example 3: Find the equation of the line passing through the points $(-5, 3)$ and $(2, 1)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 3}{2 - (-5)} = \frac{-2}{7}$$

Use $(2, 1)$:

$$y - y_1 = m(x - x_1)$$
$$y - 1 = \frac{-2}{7}(x - 2)$$
$$y - 1 = \frac{-2}{7}x + \frac{4}{7}$$
$$y = \frac{-2}{7}x + \frac{11}{7}$$

Or Use $(-5, 3)$:

$$y - y_1 = m(x - x_1)$$
$$y - 3 = \frac{-2}{7}(x - (-5))$$
$$y - 3 = \frac{-2}{7}x - \frac{10}{7}$$
$$y = \frac{-2}{7}x + \frac{11}{7}$$

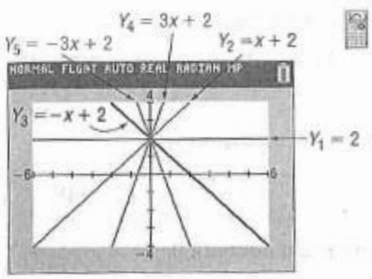


Figure 39 $y = mx + 2$

Seeing the Concept

To see the role that the slope m plays, graph the following lines on the same screen.

$Y_1 = 2 \quad m = 0$
$Y_2 = x + 2 \quad m = 1$
$Y_3 = -x + 2 \quad m = -1$
$Y_4 = 3x + 2 \quad m = 3$
$Y_5 = -3x + 2 \quad m = -3$

See Figure 39. What do you conclude about the lines $y = mx + 2$?

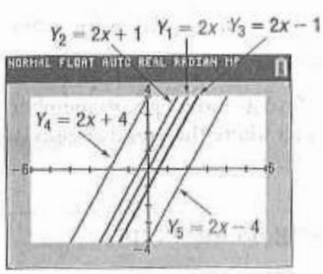


Figure 40 $y = 2x + b$

Seeing the Concept

To see the role of the y-intercept b, graph the following lines on the same screen.

$Y_1 = 2x \quad b = 0$
$Y_2 = 2x + 1 \quad b = 1$
$Y_3 = 2x - 1 \quad b = -1$
$Y_4 = 2x + 4 \quad b = 4$
$Y_5 = 2x - 4 \quad b = -4$

See Figure 40. What do you conclude about the lines $y = 2x + b$?

Slope-Intercept Form of an Equation of a Line

An equation of a line with slope m and y-intercept b is $y = mx + b$.

Example 4: Find the slope and the y-intercept of the equation $5x + 2y + 6 = 0$.

$$2y = -5x - 6$$
$$y = \frac{-5}{2}x - \frac{6}{2}$$
$$y = \frac{-5}{2}x - 3$$

$\Rightarrow$ slope $m = \frac{-5}{2}$ and y-intercept $b = -3$

Section 1.3 – Lines (continued)

General Form of a Linear Equation

The equation of a line is in **general form** when it is written as $Ax + By = C$, where A, B, and C are real numbers and A and B are not both zero.

Because the equation of every line can be written in general form, any equation equivalent to $Ax + By = C$ is called a **linear equation**.

Parallel and Perpendicular Lines

When two lines in the plane do not intersect (that is, they have no points in common), they are **parallel lines**.

Theorem: Criterion for Parallel Lines

Two nonvertical lines are parallel if and only if their slopes are equal and they have different y-intercepts.

The "if and only if" phrase means two statements are being made, one the converse of the other.

If two nonvertical lines are parallel, then their slopes are equal and they have different y-intercepts.

If two nonvertical lines have equal slopes and they have different y-intercepts, then they are parallel.

Example 5: Show that the lines $4x + 3y - 15 = 0$ and $4x + 3y + 2 = 0$ are parallel.

$4x + 3y - 15 = 0$

$3y = -4x + 15$

$y = \frac{-4}{3}x + \frac{15}{3}$

$y = \frac{-4}{3}x + 5$

$\Rightarrow$ slope $m = \frac{-4}{3}$ and y-intercept $b = 5$

$4x + 3y + 2 = 0$

$3y = -4x - 2$

$y = \frac{-4}{3}x - \frac{2}{3}$

$\Rightarrow$ slope $m = \frac{-4}{3}$ and y-intercept $b = \frac{-2}{3}$

So, these lines are parallel since they have the same slope, but different y-intercepts.

Two lines are said to be **perpendicular** if the two lines intersect at a right angle (90°).

Theorem: Criterion for Perpendicular Lines

Two nonvertical lines are perpendicular if and only if the product of their slopes is -1.

So, the slopes of non-vertical perpendicular lines are negative reciprocals of each other.

Example 6: Find an equation of a line passing through $(3, -2)$ and perpendicular to the line $x + 2y + 4 = 0$.

Given $x + 2y + 4 = 0$.

$2y = -x - 4$

$y = \frac{-1}{2}x - 2$

$\Rightarrow$ Given slope: $m = \frac{-1}{2}$

New perpendicular slope: $m_{new} = 2$.

So, you want a line through $(3, -2)$ with slope $m = 2$.

$y - y_1 = m(x - x_1)$

$y - (-2) = 2(x - 3)$

$y + 2 = 2x - 6$

Thus, $y = 2x - 8$ or $2x - y - 8 = 0$.

All material has been taken from *Precalculus,* by M. Sullivan, 10th Edition