Section 1.4 – Circles

Circles

A **circle** is a set of points in the xy-plane that are a fixed distance *r* from a fixed point (h, k). The fixed distance *r* is called the **radius**, and the fixed point (h, k) is called the **center** of the circle.

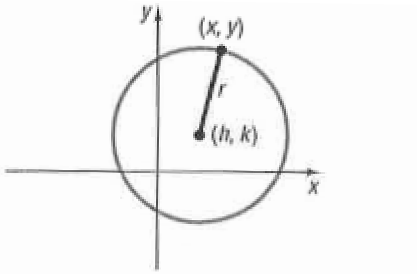


Figure 49 $(x-h)^2+(y-k)^2=r^2$

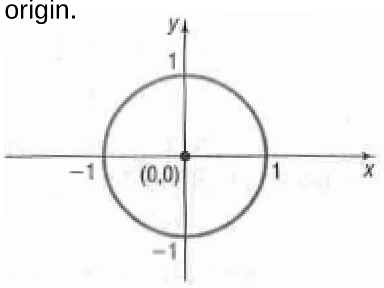
Figure 49 shows the graph of a circle. To find the equation, let (x, y) represent the coordinates of any point on a circle with radius *r* and center (h, k). Then the distance between the points (x, y) and (h, k) must always equal r. That is, by the distance formula, $\sqrt{(x-h)^2+(y-k)^2}=r$, or equivalently, $(x-h)^2+(y-k)^2=r^2$.

The **standard form of an equation of a circle** with radius *r* and center (h, k) is $(x-h)^2+(y-k)^2=r^2$.

Theorem: The standard form of an equation of a circle of radius *r* with center at the origin (0, 0) is

$$x^2+y^2=r^2.$$

If the radius $r=1$, the circle whose center is at the origin is called the **unit circle** and has the equation $x^2+y^2=1$. From the figure below, you can see the unit circle is symmetric with respect to the x-axis, the y-axis, and the origin.



Example 1: Write the standard form of the equation of the circle with radius 4 and center $(2,-5)$.
Substitute the values $r=4$, $h=2$, and $k=-5$ into the circle equation.

$$(x-h)^2+(y-k)^2=r^2$$
$$(x-2)^2+(y-(-5))^2=4^2$$
$$(x-2)^2+(y+5)^2=16$$

Section 1.4 – Circles (continued)

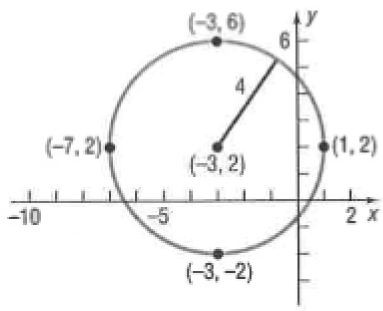
Graphing a Circle

Example 2: Graph the equation: $(x+3)^2 + (y-2)^2 = 16$.

Since the equation is in the standard form of the equation of a circle, its graph is a circle. To graph the equation, compare the given equation to the standard form of the equation of a circle. The comparison gives you information about the circle.

$$(x+3)^2 + (y-2)^2 = 16$$
$$(x-(-3))^2 + (y-2)^2 = 4^2$$
$$(x-h)^2 + (y-k)^2 = r^2$$

You can see that $h = -3,\ k = 2,$ and $r = 4$. Thus, the circle has its center at $(h, k) = (-3, 2)$ and a radius of 4 units. You can graph four points on the circle by plotting points 4 units to the left, to the right, up, and down from the center. Now draw a circle through these four points.



Example 3: Find the intercepts, if any, of the graph of the circle $(x+2)^2 + (y+1)^2 = 25$.

To find the x-intercepts, set y=0.

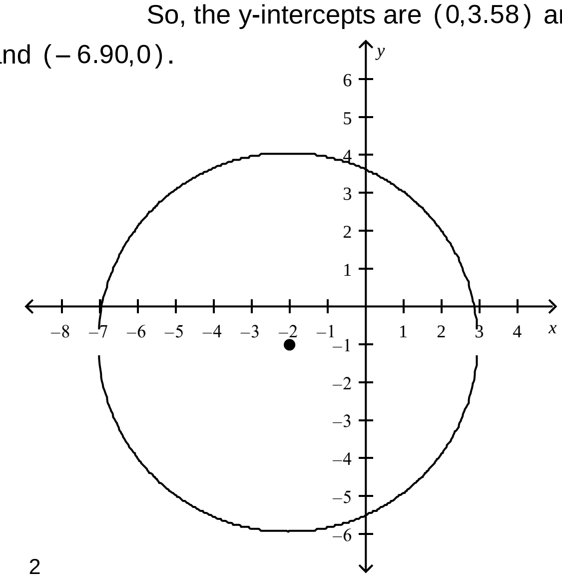
$$(x+2)^2 + (0+1)^2 = 25$$
$$(x+2)^2 + (1)^2 = 25$$
$$(x+2)^2 + 1 = 25$$
$$(x+2)^2 = 24$$
$$x+2 = \pm\sqrt{24}$$
$$x = -2 \pm \sqrt{4}\sqrt{6}$$
$$x = -2 \pm 2\sqrt{6}$$
$$x = -2+2\sqrt{6} \quad \text{and} \quad x = -2-2\sqrt{6}$$
$$x \approx 2.898 \quad \text{and} \quad x = -6.898$$

So, the x-intercepts are $(2.90, 0)$ and $(-6.90, 0)$.

To find the y-intercepts, set x=0.

$$(0+2)^2 + (y+1)^2 = 25$$
$$(2)^2 + (y+1)^2 = 25$$
$$4 + (y+1)^2 = 25$$
$$(y+1)^2 = 21$$
$$y+1 = \pm\sqrt{21}$$
$$y = -1 \pm \sqrt{21}$$
$$y = -1+\sqrt{21} \quad \text{and} \quad y = -1-\sqrt{21}$$
$$y \approx 3.582 \quad \text{and} \quad y = -5.582$$

So, the y-intercepts are $(0, 3.58)$ and $(0, -5.58)$.



Section 1.4 – Circles (continued)

Work with the General Form of the Equation of a Circle

When its graph is a circle, the equation $x^2 + y^2 + ax + by + c = 0$ is the **general form of the equation of a circle**.

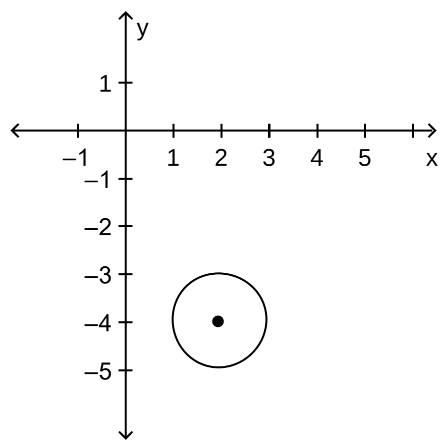
If an equation of a circle is in general form, use the method of completing the square to put the equation in standard form in order to identify its center and radius.

Example 4: Rewrite $x^2 + y^2 - 4x + 8y + 19 = 0$ in standard form and graph the equation.

The equation is given in general form. Rewrite the equation by grouping the terms involving x, grouping the terms involving y, and putting the constant on the right side of the equation. Next, complete the square. Remember that any number added to the left side must also be added to the right side.

$$x^2 + y^2 - 4x + 8y + 19 = 0$$
$$x^2 - 4x + y^2 + 8y = -19$$
$$x^2 - 4x + 4 + y^2 + 8y + 16 = -19 + 4 + 16$$
$$(x-2)^2 + (y+4)^2 = 1$$
$$(x-h)^2 + (y-k)^2 = r^2$$

This equation is the standard form of the equation of a circle with center $(h, k) = (2, -4)$ and radius $r = 1$. To graph the equation, use the center and the radius.

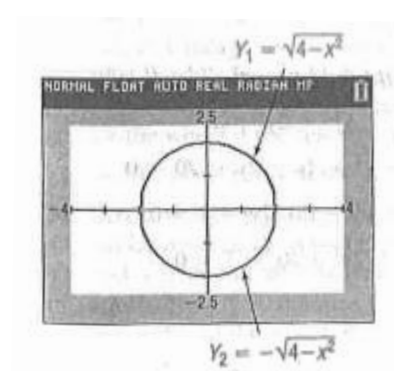


Using a Graphing Utility to Graph a Circle

Example 5: Graph the equation $x^2 + y^2 = 4$.

This is the equation of a circle with center at the origin and radius 2. To graph the circle on your calculator, solve for y.

$$x^2 + y^2 = 4$$
$$y^2 = 4 - x^2$$
$$y = \pm\sqrt{4 - x^2}$$



There are two equations to graph on the same square screen:
$Y_1 = \sqrt{4 - x^2}$ and $Y_2 = -\sqrt{4 - x^2}$

Your circle will appear oval if you do not use a square screen.

All material has been taken from *Precalculus,* by M. Sullivan, 10th Edition