Section 2.4 – Library of Functions; Piecewise-defined Functions

Library of Functions

You have encountered many functions, some of which have special names. They each have special characteristics, including the shape of their graph. You should be familiar with these functions and their graphs. When you encounter one of these library functions, you should see in your mind's eye a figure like those pictured below.

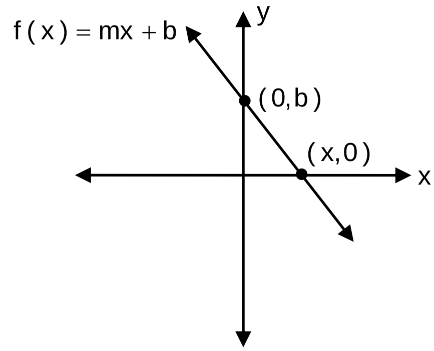
Linear Functions:

$f(x) = mx + b$, where m and b are real numbers.

Domain: All real numbers

Graph: A nonvertical line with slope m and y-intercept b.

Increasing if $m > 0$, decreasing if $m < 0$, and constant if $m = 0$.



Constant Function:

$f(x) = b$, where b is a real number.

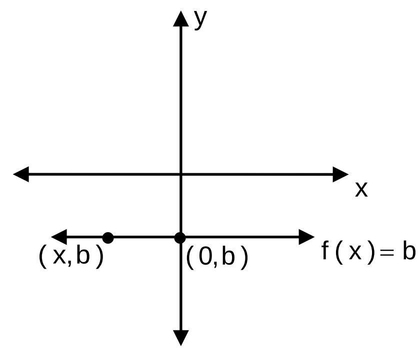
This is a special linear function where slope $m = 0$.

Domain: All real numbers

Range: The single number b

Graph: A horizontal line whose y-intercept is b.

It is an even function (sym wrt the y-axis) whose graph is constant over its domain.



Identity Function:

$f(x) = x$

This is a special linear function where the slope $m = 1$ and the y-intercept $b = 0$.

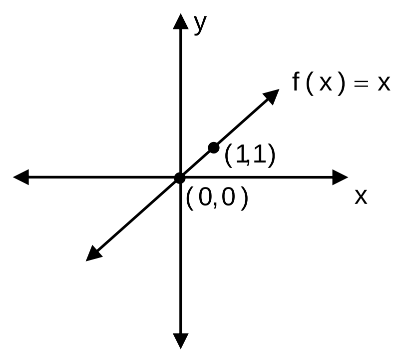
Domain: All real numbers

Range: All real numbers

Graph: A line of all points for which the x-coordinate equals the y-coordinate.

It is an odd function (sym wrt the origin) that is increasing over its domain.

The graph bisects quadrants I and III.



Square Function:

$f(x) = x^2$

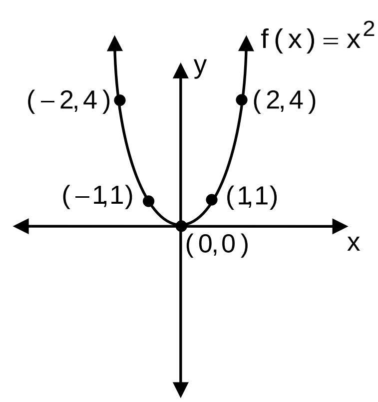
Domain: All real numbers

Range: All nonnegative real numbers

Graph: A parabola whose intercept (x and y) is at $(0, 0)$.

It is an even function (sym wrt the y-axis) that is decreasing on the interval $(-\infty, 0)$ and increasing on the interval $(0, \infty)$.

It has an absolute minimum of 0 at $x = 0$.



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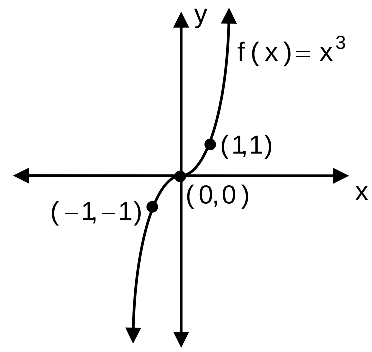
Cube Function:

$f(x)=x^3$

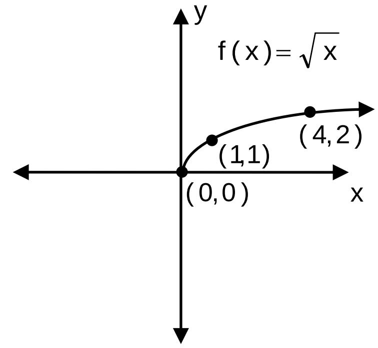
Domain: All real numbers
Range: All real numbers
Graph: It has an intercept (x and y) at $(0,\ 0)$.
It is an odd function (sym wrt the origin) that is increasing on the interval $(-\infty,\ \infty)$.
It does not have any local minima or any local maxima.



Square Root Function:

$f(x)=\sqrt{x}$

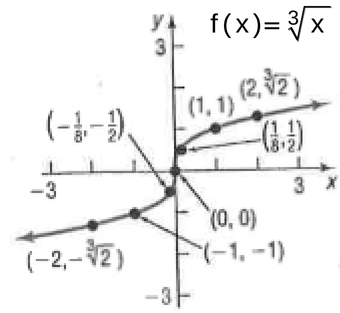
Domain: All nonnegative real numbers
Range: All nonnegative real numbers
Graph: It has an intercept (x and y) at $(0,\ 0)$.
It is neither even nor odd, and it is increasing on the interval $(0,\infty)$.
It has an absolute minimum of 0 at $x=0$.



Cube Root Function:

$f(x)=\sqrt[3]{x}$

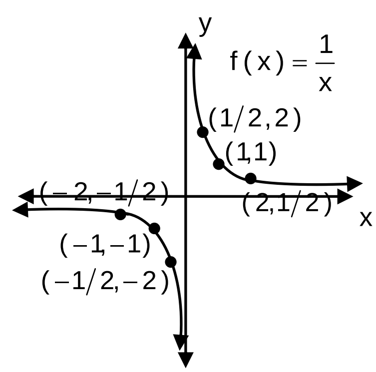
Domain: All real numbers
Range: All real numbers
Graph: It has an intercept (x and y) at $(0,\ 0)$.
It is an odd function (sym wrt the origin), and it is increasing on the interval $(-\infty,\ \infty)$.
It does not have any local minima or any local maxima.



Reciprocal Function:

$f(x)=\dfrac{1}{x}$

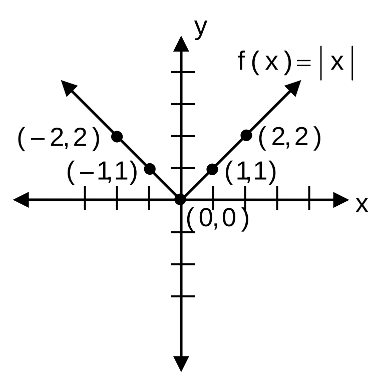
Domain: All real numbers except 0
Range: All real numbers except 0
Graph: It has no intercepts.
It is an odd function (sym wrt the origin), and it is decreasing on the intervals $(-\infty,\ 0)$ and $(0,\ \infty)$.



Absolute Value Function:

$f(x)=|x|$

Domain: All real numbers
Range: All nonnegative real numbers
Graph: It has an intercept (x and y) at $(0,\ 0)$.
It is an even function (sym wrt the y-axis), and it is decreasing on the interval $(-\infty,\ 0)$ and increasing on the interval $(0,\ \infty)$.
It has an absolute minimum of 0 at $x=0$.



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Notation: $\text{int}(x)$ means the largest integer less than or equal to x.

So, for example, $\text{int}(7)=7$, $\text{int}(3.5)=3$, and $\text{int}\left(\frac{-5}{6}\right)=-1$.

Greatest Integer Function:

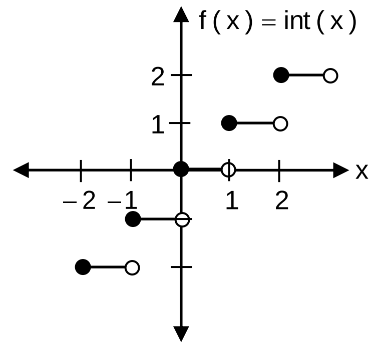
$f(x)=\text{int}(x)=$ Greatest integer less than or equal to x

Domain: All real numbers
Range: All integers
Graph: y-intercept at $(0, 0)$.

x-intercepts lie on the interval $[0, 1)$.

It is constant on the interval $[k, k+1)$ for k an integer.

It is neither an even nor an odd function.
It is also called a step function.
At $x=0$, $x=\pm1$, $x=\pm2$, … there is a discontinuity; that is, at integer values, the graph suddenly "steps" from one value to another without taking on any of the intermediate values.



When graphing the greatest integer function on your calculator, you want to set the mode to dot mode (instead of connected mode) to prevent the calculator from "connecting the dots" when $f(x)$ changes from one integer value to the next.

Piecewise-defined Functions

Sometimes a function is defined using different equations on different parts of its domain. When a function is defined by different equations on different parts of its domain, it is called a piecewise-defined function.

Example 1: The absolute value function $f(x)=|x|$ is a piecewise-defined function. It is actually defined by two equations: $f(x)=x$ if $x\ge 0$ and $f(x)=-x$ if $x<0$. You combine these into one expression as

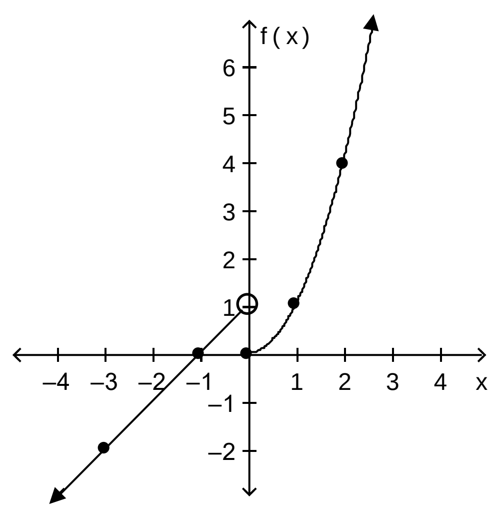
$$f(x)=|x| = \begin{cases} x & \text{if } x\ge 0 \\ -x & \text{if } x<0 \end{cases}$$

The graph of $f(x)$ looks like the graph on the bottom of page 2.

Example 2: Use a T-table to graph the function $f(x)$.
Find the domain and range of $f(x)$.

$$f(x)=\begin{cases} 1+x & \text{if } x<0 \\ x^2 & \text{if } x\ge 0 \end{cases}$$

	x	f(x)
First piece $x<0$	-3	-2
	-2	-1
	-1	0
	$-\frac{1}{4}$	$\frac{3}{4}$
Second piece $x\ge 0$	0	0
	1	1
	2	4



Domain: All real numbers
Range: All real numbers

A function is **discontinuous** if its graph has holes or gaps and so cannot be drawn without lifting a pencil from the paper. Example 2 is discontinuous.

All material has been taken from *Precalculus,* by M. Sullivan, 10th Edition