Section 2.5 – Graphing Techniques: Transformations – Day 1

Transformations are techniques that alter the graphs of the basic library functions.

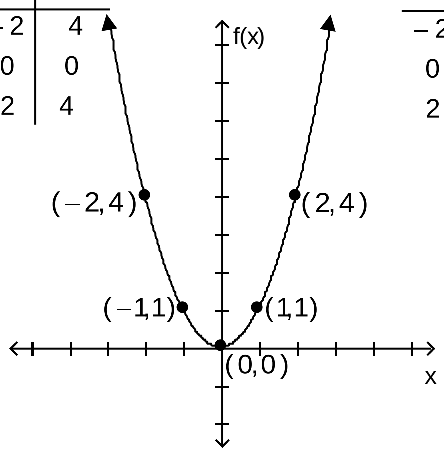
Vertical Shifts

If a real number k is added to the right side of a function $y = f(x)$, the graph of the new function $y = f(x) + k$ is the graph of f shifted vertically up k units (if $k > 0$) or down k units (if $k < 0$).

Example 1: Given $f(x) = x^2$, graph $g(x) = f(x) + 3$ and $h(x) = f(x) - 4$.

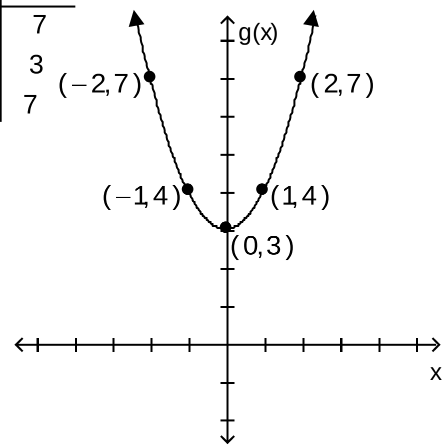
$f(x) = x^2$, Library/Basic function

x	f(x)
−2	4
0	0
2	4



$g(x) = f(x) + 3$
$= x^2 + 3$
Shift up 3 units

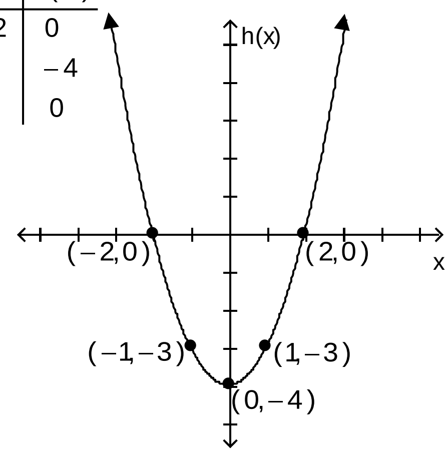
x	g(x)
−2	7
0	3
2	7



and

$h(x) = f(x) - 4$
$= x^2 - 4$
Shift down 4 units

x	h(x)
−2	0
0	−4
2	0



Note that a vertical shift affects only the range of a function, not the domain. In Example 1, the range of $f(x)$ is $[0,\infty)$, the range of $g(x)$ is $[3,\infty)$, and the range of $h(x)$ is $[-4,\infty)$. The domain for all three functions is all real numbers. In a vertical shift, the x-coordinate of a point does not change, but the y-coordinate increases, shifting the graph up if $k > 0$, or decreases, shifting the graph down if $k < 0$.

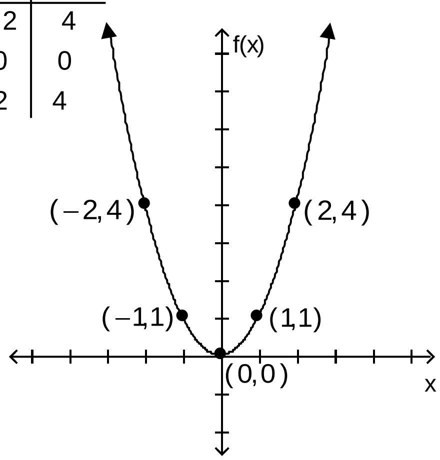
Horizontal Shifts

If the argument x of a function f is replaced by $x - h$, where h is a real number, then the graph of the new function $y = f(x - h)$ is the graph of f shifted horizontally left (if $h < 0$) or right (if $h > 0$).

Example 2: Given $f(x) = x^2$, graph $g(x) = f(x - 3)$ and $h(x) = f(x + 5)$.

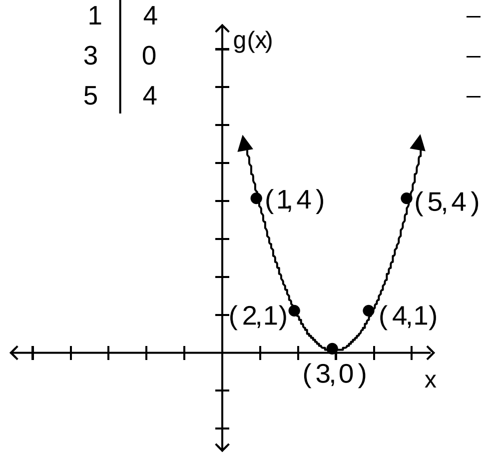
$f(x) = x^2$, Library/Basic function

x	f(x)
−2	4
0	0
2	4



$g(x) = f(x - 3)$
$= (x - 3)^2$
Shift right 3 units

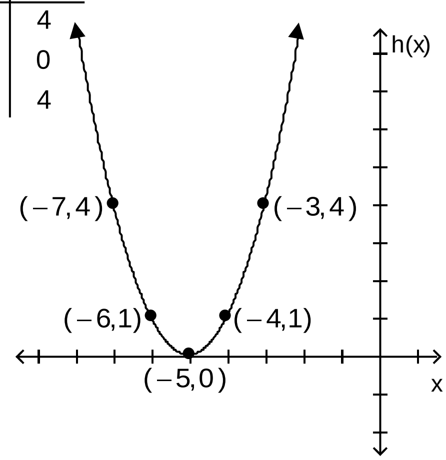
x	g(x)
1	4
3	0
5	4



and

$h(x) = f(x + 5)$
$= (x + 5)^2$
Shift left 5 units

x	h(x)
−7	4
−5	0
−3	4



Note that a horizontal shift affects only the domain of a function, not the range.

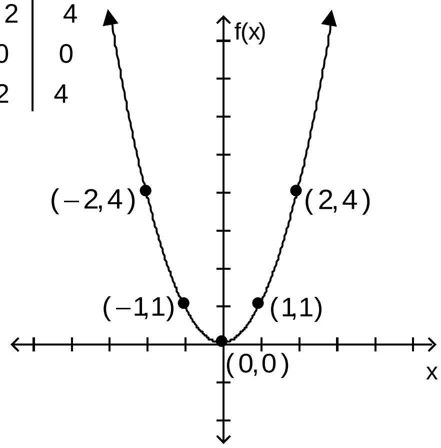
Section 2.5 – Graphing Techniques: Transformations – Day 1 (continued)

You can combine vertical and horizontal shifts.

Example 3: Graph $h(x) = (x-2)^2 + 4$.

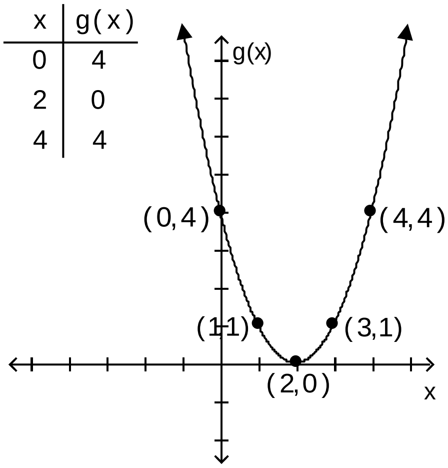
1) $f(x) = x^2$
Library/Basic function

x	f(x)
−2	4
0	0
2	4



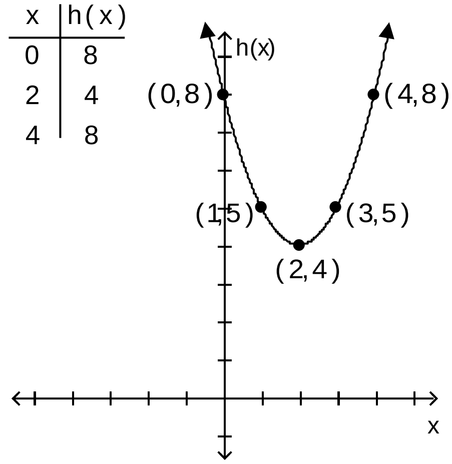
2) $g(x) = f(x-2)$
$= (x-2)^2$
Shift right 2 units

x	g(x)
0	4
2	0
4	4



3) $h(x) = g(x) + 4$
$= (x-2)^2 + 4$
Shift up 4 units

x	h(x)
0	8
2	4
4	8



Note the points plotted on each graph. Using key points can be helpful in keeping track of the transformation that has taken place.

In Example 3, if the vertical shift had been done first, followed by the horizontal shift, the final graph would have been the same.

Compressions and Stretches

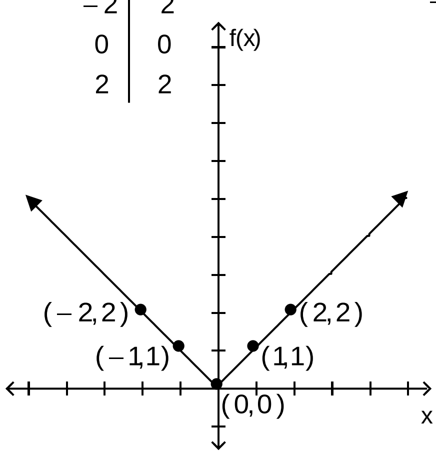
Vertical Compression/Stretch:

When the right side of a function $y = f(x)$ is multiplied by a positive number b, the graph of the new function $y = bf(x)$ is obtained by multiplying each y-coordinate of $y = f(x)$ by b. A vertical compression by a factor of b results if $0 < b < 1$ and a vertical stretch by a factor of b occurs if $b > 1$.

Example 4: Given $f(x) = |x|$, graph $g(x) = 3f(x)$ and $h(x) = \frac{1}{2}f(x)$.

$f(x) = |x|$
Library/ Basic function

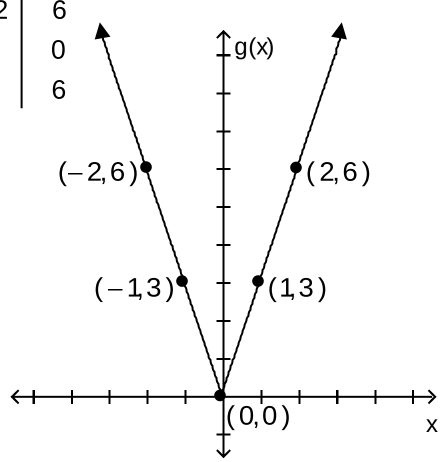
x	f(x)
−2	2
0	0
2	2



$g(x) = 3f(x)$
$= 3|x|$
Vertical stretch by a factor of 3

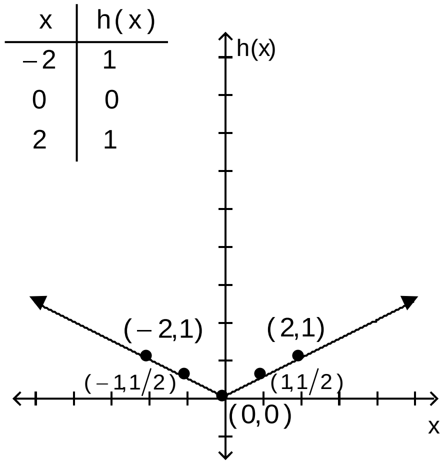
and

x	g(x)
−2	6
0	0
2	6



$h(x) = \frac{1}{2}f(x)$
$= \frac{1}{2}|x|$
Vertical compression by a factor of $\frac{1}{2}$

x	h(x)
−2	1
0	0
2	1



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Horizontal Compression/Stretch:

If the argument x of a function $y = f(x)$ is multiplied by a positive number b, the graph of the new function $y = f(bx)$ is obtained by multiplying each x-coordinate of $y = f(x)$ by $\frac{1}{b}$. A horizontal compression by a factor of $\frac{1}{b}$ results if $b > 1$ and a horizontal stretch by a factor of $\frac{1}{b}$ occurs if $0 < b < 1$.

Example 5: Given $f(x) = \sqrt{16 - x^2}$, graph $g(x) = f(2x)$ and $h(x) = f\left(\frac{1}{2}x\right)$.

$f(x) = \sqrt{16 - x^2}$

Library/Basic function

x	f(x)
−4	0
−2	$\sqrt{12}$
0	4
2	$\sqrt{12}$
4	0

$$g(x) = f(2x) = \sqrt{16 - (2x)^2} = \sqrt{16 - 4x^2}$$

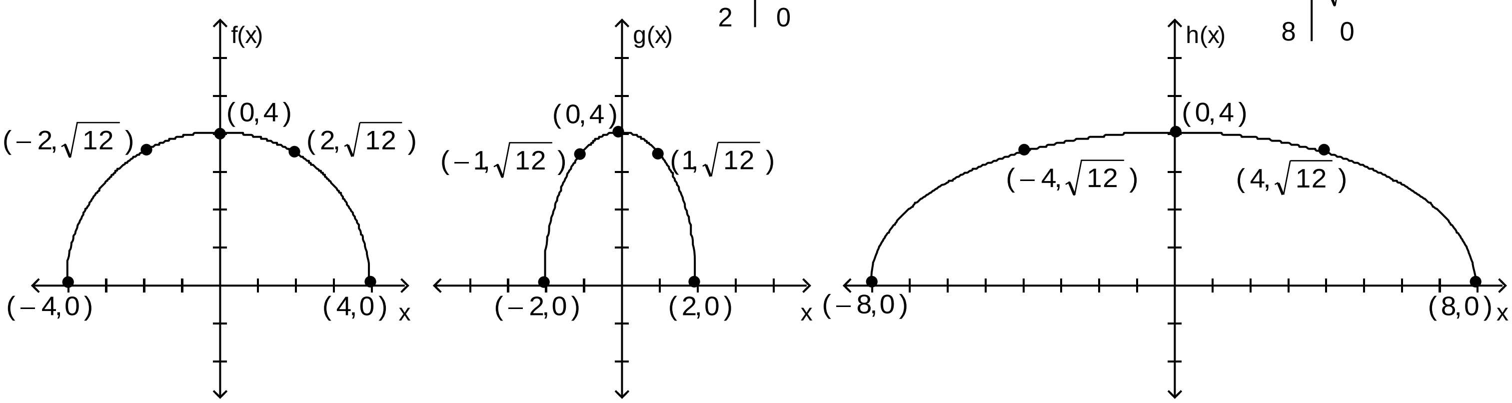
Horizontal compression by a factor of $\frac{1}{2}$

x	g(x)
−2	0
−1	$\sqrt{12}$
0	4
1	$\sqrt{12}$
2	0

$$h(x) = f\left(\frac{1}{2}x\right) = \sqrt{16 - \left(\frac{1}{2}x\right)^2} = \sqrt{16 - \frac{x^2}{4}}$$

Horizontal stretch by a factor of 2

x	h(x)
−8	0
−4	$\sqrt{12}$
0	4
4	$\sqrt{12}$
8	0



Also, $f(1) = \sqrt{15} \approx 3.87$, $f(2) = \sqrt{12} \approx 3.46$, $f(3) = \sqrt{7} \approx 2.65$

$g(1) = \sqrt{12} \approx 3.46$, $g(1.5) = \sqrt{7} \approx 2.65$

$h(2) = \sqrt{15} \approx 3.87$, $h(4) = \sqrt{12} \approx 3.46$, $h(6) = \sqrt{7} \approx 2.65$

Domain of $f = \{x \mid -4 \le x \le 4\}$ Domain of $g = \{x \mid -2 \le x \le 2\}$ Domain of $h = \{x \mid -8 \le x \le 8\}$

When graphing transformations, do the following:

1) Give separate graphs for each step in the transformation.
2) State the function, particularly in terms of the previous function.
3) Describe the transformation or shift.
4) Follow at least three points on each branch of the graph. Label these points or list them, with increasing values of x, in a table.