## Section 3.3 – Quadratic Functions and Their Properties – Day 2

A quadratic function is a function of the form $f(x) = ax^2 + bx + c$, where a, b, and c are real numbers and $a \neq 0$. So, a quadratic function is a function defined by a second-degree polynomial in one variable.

From algebra, you know the following for a quadratic function $f(x) = ax^2 + bx + c$:

1) The graph of $f(x)$ is a parabola.

2) If $a > 0$, the parabola opens upward; the vertex is a minimum point.
   If $a < 0$, the parabola opens downward; the vertex is a maximum point.

3) The highest or lowest (maximum or minimum) point of a parabola is called the vertex.
   The vertex has the coordinate point $\left( \frac{-b}{2a}, f\left( \frac{-b}{2a} \right) \right)$.

4) The vertical line passing through the vertex of a parabola is called the axis of symmetry (AOS).
   The axis of symmetry is the vertical line $x = \frac{-b}{2a}$.

5) The y-intercept is $(0, c)$, since $f(0) = c$.

6) By completing the square, we can write $f(x)$ in the form $f(x) = a(x - h)^2 + k$, where (h, k) is the vertex of the parabola and the axis of symmetry is the vertical line $x = h$.
   a represents the vertical stretch or compression.
   h represents the horizontal shift.
   k represents the vertical shift.

7) The x-intercepts, if there are any, are found by solving the equation $f(x) = 0$, i.e., $ax^2 + bx + c = 0$.
   This equation has two, one, or no real solutions, depending on whether the discriminant $b^2 - 4ac$ is positive, zero, or negative.
   The graph of $f(x)$ has two distinct x-intercepts if $b^2 - 4ac > 0$.
   => The graph crosses the x-axis in two places.

   The graph of $f(x)$ has one x-intercept if $b^2 - 4ac = 0$.
   => The graph touches the x-axis at its vertex.

   The graph of $f(x)$ has no x-intercepts if $b^2 - 4ac < 0$.
   => The graph does not cross or touch the x-axis.

   Figure 18 illustrates these possibilities for parabolas that open upward.

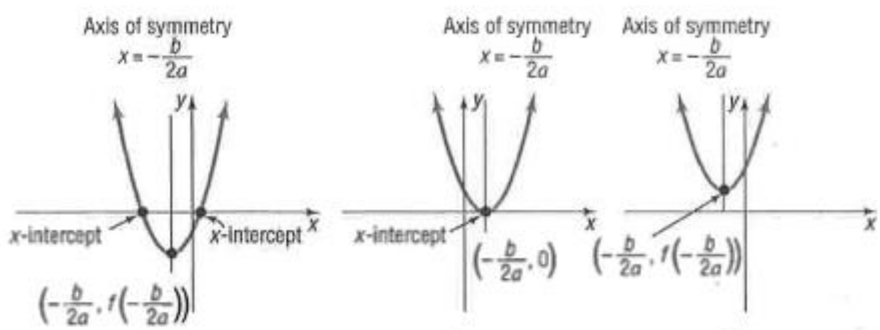



**Figure 18** $f(x) = ax^2 + bx + c,\ a > 0$
(a) $b^2 - 4ac > 0$ Two x-intercepts
(b) $b^2 - 4ac = 0$ One x-intercept
(c) $b^2 - 4ac < 0$ No x-intercepts

Section 3.3 – Quadratic Functions and Their Properties – Day 2 (continued)

So, there are two methods of graphing a quadratic function:

1) Complete the square and apply shifting techniques (transformations).
2) Use the vertex and axis of symmetry formulas, determine whether the graph opens upward or downward, and locate the intercepts, if there are any intercepts.

Example 2: Graph the quadratic function $f(x) = -2x^2 + 4x + 1$ using its vertex, axis of symmetry, and intercepts.

$$f(x) = -2x^2 + 4x + 1$$
$$= ax^2 + bx + c$$
$$\Rightarrow a = -2,\ b = 4,\ c = 1$$

$$x_v = \frac{-b}{2a} = \frac{-4}{2(-2)} = \frac{-4}{-4} = 1$$

$$f(x_v) = f(1) = -2(1)^2 + 4(1) + 1 = -2(1) + 4 + 1 = -2 + 5 = 3$$

The vertex: $(x_v, f(x_v)) = (1, 3)$

Axis of symmetry: the line $x = x_v$

$$x = 1$$

Direction of opening: $a = -2$

$a < 0 \Rightarrow$ opens downward

y-intercept: set $x = 0$

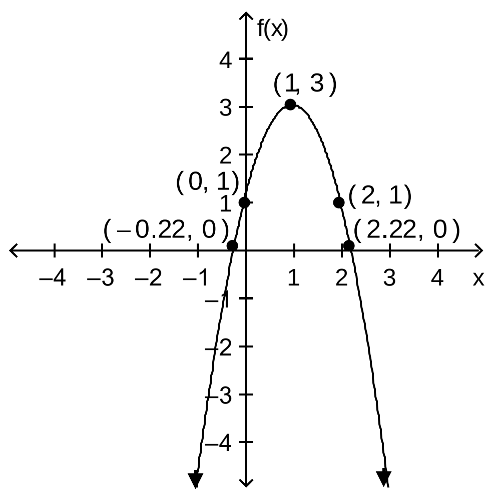
$$f(0) = -2(0)^2 + 4(0) + 1 = 0 + 0 + 1 = 1$$

y-intercept: $(0, 1)$

x-intercept: set $f(x) = 0$

$$-2x^2 + 4x + 1 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-4 \pm \sqrt{4^2 - 4(-2)(1)}}{2(-2)}$$
$$= \frac{-4 \pm \sqrt{16 + 8}}{-4}$$
$$= \frac{-4 \pm \sqrt{24}}{-4}$$
$$= \frac{-4 \pm \sqrt{4}\sqrt{6}}{-4}$$
$$= \frac{-4 \pm 2\sqrt{6}}{-4}$$
$$= \frac{2(-2 \pm \sqrt{6})}{2(-2)}$$
$$= \frac{-2 \pm \sqrt{6}}{-2}$$
$$\approx -0.2247 \text{ or } 2.2247$$

x-intercepts: $(-0.22, 0)$ and $(2.22, 0)$




The function is decreasing on the interval $(1, \infty)$.

The function is increasing on the interval $(-\infty, 1)$.

## Section 3.3 – Quadratic Functions and Their Properties – Day 2

You do not need to complete the square to obtain the vertex. It is often easier to obtain the vertex of a quadratic function f by remembering that its x-coordinate is $x_v = \frac{-b}{2a}$. The y-coordinate can then be found by evaluating function f at $\frac{-b}{2a}$, that is find $f\left(\frac{-b}{2a}\right)$.

Example 3: Without graphing, determine the vertex, the axis of symmetry, and the direction of opening of the parabola defined by $f(x) = -3x^2 + 12x + 4$.

$$x_v = \frac{-b}{2a} = \frac{-12}{2(-3)} = \frac{-12}{-6} = 2$$

$$f(x_v) = f(2) = -3(2)^2 + 12(2) + 4 = -3(4) + 24 + 4 = -12 + 28 = 16$$

The vertex: $(x_v, f(x_v)) = (2, 16)$

Axis of symmetry: the line $x = x_v$, $x = 2$

Direction of opening: $a = -3$, $a < 0 \Rightarrow$ opens downward

If the graph of a quadratic has only one x-intercept or no x-intercepts, it is usually necessary to plot an additional point to obtain the graph.

If the vertex $(h, k)$ and one additional point on the graph of a quadratic function $f(x) = ax^2 + bx + c, a \neq 0$, are known, then $f(x) = a(x-h)^2 + k$ can be used to obtain the quadratic function.

Example 4: Determine the quadratic function whose vertex is $(2,5)$ and whose y-intercept is $(0,-3)$.

$f(x) = a(x-h)^2 + k$

The vertex is $(h,k) = (2,5)$, so $h = 2$ and $k = 5$. Substituting these values into the function, you get

$$f(x) = a(x-2)^2 + 5.$$

The y-intercept is $(0,-3)$, so $f(0) = -3$.

$$f(0) = -3$$
$$a(0-2)^2 + 5 = -3$$
$$a(-2)^2 + 5 = -3$$
$$4a + 5 = -3$$
$$4a = -8$$
$$a = -2$$

So, the function is $f(x) = -2(x-2)^2 + 5$ or

$$f(x) = -2(x-2)^2 + 5 = -2(x^2 - 4x + 4) + 5 = -2x^2 + 8x - 8 + 5 = -2x^2 + 8x - 3$$