Section 4.1 – Polynomial Functions and Models – Day 1

Identify Polynomial Functions and Their Degree

A polynomial function in one variable is a function of the form $f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$, where $a_n, a_{n-1}, \ldots, a_1, a_0$ are constants, called the **coefficients** of the polynomial, $n \geq 0$ is an integer, and *x* is the variable. If $a_n \neq 0$, it is called the **leading coefficient**, and *n* is the **degree** of the polynomial.
The domain of a polynomial function is the set of all real numbers.

The monomials that make up a polynomial are called its **terms**. If $a_n \neq 0$, $a_n x^n$ is called the **leading term**; a_0 is called the **constant term**. If all of the coefficients are 0, the polynomial is called the **zero polynomial**, which has no degree.

Polynomials are usually written in **standard form**, beginning with the nonzero term of highest degree and continuing with terms in descending order according to degree. If a power of x is missing, it is because its coefficient is zero.

Polynomial functions are among the simplest expressions in algebra. They are easy to evaluate: only addition and repeated multiplication are required. Because of this, they are often used to approximate other, more complicated functions. In this section, you will investigate properties of this important class of functions.

The degree of a polynomial function is the degree of the polynomial in one variable, i.e., the largest power of x.

Example 1: Are the following polynomial functions? If yes, state the degree and identify the leading term and the constant term. If not, state why not.

a) $f(x) = 2x^5 - 3x^2$ Yes, a polynomial of degree 5. The leading term is $2x^5$, and the constant term is 0.

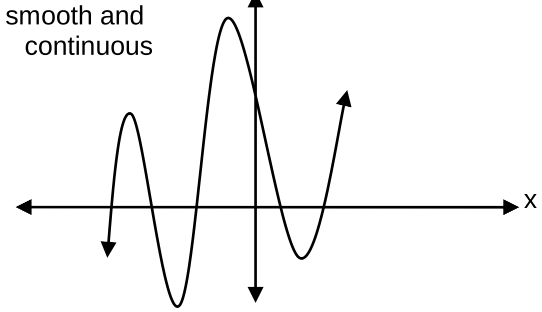
b) $g(x) = 6$ Yes or No ? Why/Why not?

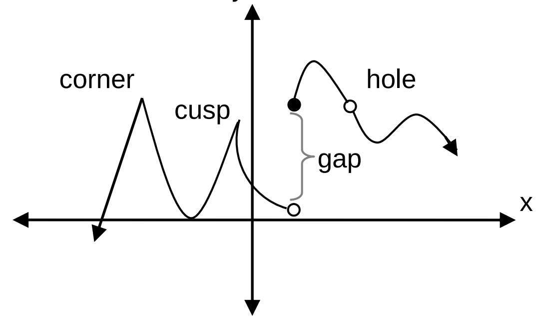
c) $h(x) = \sqrt{x} + 4$ Yes or No ? Why/Why not?

Characteristics of Basic Polynomial Functions –

Degree	Form	Name	Graph
No degree	$f(x) = 0$	Zero Function	The x-axis
0	$f(x) = a_0,\ a_0 \neq 0$	Constant Function	A horizontal line with a y-intercept at $(0, a_0)$
1	$f(x) = a_1 x + a_0,\ a_1 \neq 0$	Linear Function	A non-vertical / non-horizontal line with slope a_1 and y-intercept at $(0, a_0)$
2	$f(x) = a_2 x^2 + a_1 x + a_0,\ a_2 \neq 0$	Quadratic Function	A parabola: opens upward if $a_2 > 0$, opens downward if $a_2 < 0$

One objective of this section is to analyze the graph of a polynomial function. If you take a course in calculus, you will learn that the graph of every polynomial is both smooth and continuous. **Smooth** implies the graph contains no sharp corners or cusps. **Continuous** implies the graph has no gaps or holes and can be drawn without lifting your pencil from the paper.





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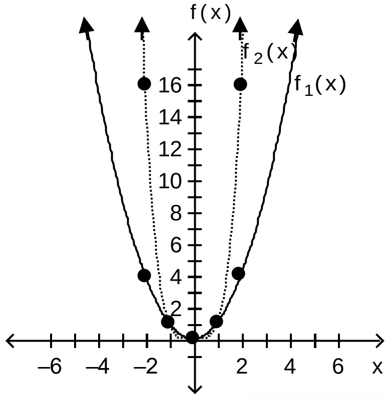
Power Functions

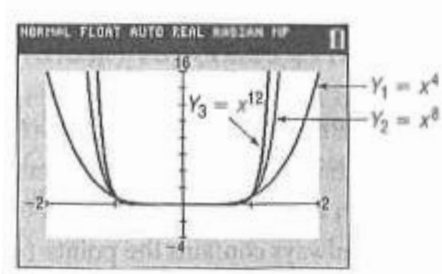
A **power function of degree n** is a monomial function of the form $f(x) = ax^n$, where *a* is a real number, $a \neq 0$, and $n > 0$ is an integer.

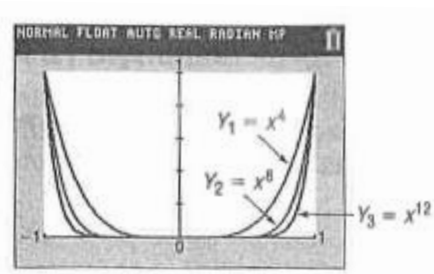
Consider $f(x) = x^n$, where n is even and $n \geq 2$. $f(x)$ is an even function, so its graph is symmetric with respect to the y-axis. Domain: All real numbers. Range: All nonnegative real numbers.

Let $f_1(x) = x^2$, $f_2(x) = x^4$, and $f_3(x) = x^8$. Graphs? As the power of x^n increases, x^n tends to _____________ near the origin ($-1 < x < 1$) and is steeper when x is far from 0 (it becomes more vertical). For large n, it may appear that the graph coincides with the x-axis near the origin, but it does not; the graph actually touches the x-axis only at the origin. Also, for large n, it may appear that for $x < -1$ or for $x > 1$ the graph is vertical, but it is not; it is only increasing very rapidly in these intervals.
The graph always contains the points $(-1, 1)$, $(0, 0)$, and $(1, 1)$.



x	$f_1(x) = x^2$	$f_2(x) = x^4$	$f_3(x) = x^8$
−3			
−2			
−1			
0			
1			
2			
3			

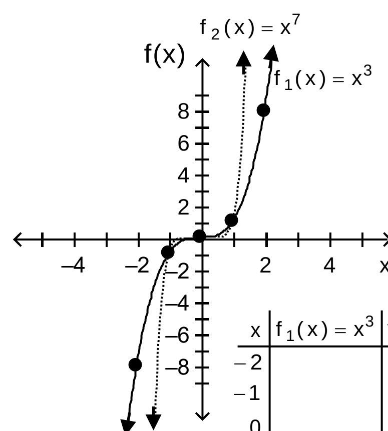




Properties of Power Functions, $f(x) = x^n$, n Is a Positive Even Integer

1) f is an even function, so its graph is symmetric with respect to the y-axis.
2) The domain is the set of all real numbers. The range is the set of nonnegative real numbers.
3) The graph always contains the points $(-1, 1)$, $(0, 0)$, and $(1, 1)$.
4) As the exponent n increases in magnitude, the graph is steeper when $x < -1$ or $x > 1$; but for x near the origin, the graph tends to flatten out and lie closer to the x-axis.

Now consider $f(x) = x^n$, where n is odd and $n \geq 3$. $f(x)$ is an odd function, so its graph is symmetric with respect to the origin. Domain: All real numbers. Range: All real numbers.



x	$f_1(x) = x^3$	$f_2(x) = x^7$
−2		
−1		
0		
1		
2		

As the power of x^n increases, x^n tends to _______________ near the origin ($-1 < x < 1$) and increases rapidly when x is far from 0 (it becomes more vertical). The graph always contains the points $(-1, -1)$, $(0, 0)$, and $(1, 1)$.

It appears that each graph coincides with the x-axis near the origin, but it does not; each graph actually crosses the x-axis at the origin. Also, it appears that as x increases the graphs become vertical, but they do not; each graph is just increasing very rapidly.

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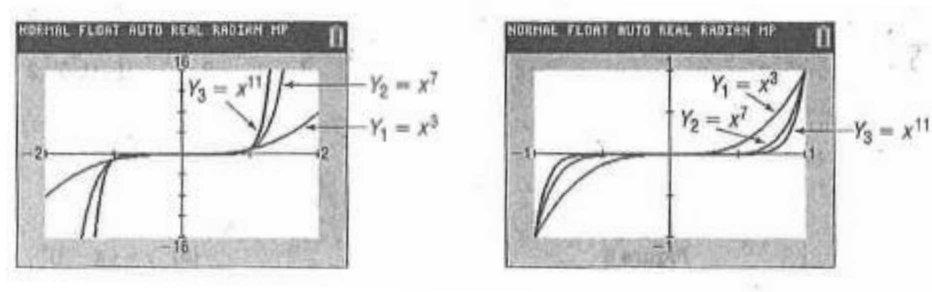

Properties of Power Functions, $f(x) = x^n$, n Is a Positive Odd Integer

1) f is an odd function, so its graph is symmetric with respect to the origin.
2) The domain and the range are the set of all real numbers.
3) The graph always contains the points $(-1,-1)$, $(0, 0)$, and $(1, 1)$.
4) As the exponent n increases in magnitude, the graph is steeper when $x < -1$ or $x > 1$; but for x near the origin, the graph tends to flatten out and lie closer to the x-axis.

Graph Polynomial Functions Using Transformations

Using transformations (shifting, compressing, stretching, and reflecting) and the facts presented above, enables you to graph polynomial functions that are transformations of power functions.

Example 2: Graph $h(x) = 2 - x^3$.

a) $f(x) = x^3$ Basic function

b) $g(x) = -f(x)$ Reflect about the x-axis (or y-axis if $f(-x)$)
$= -x^3$

c) $h(x) = g(x) + 2$ Shift up 2 units
$= -x^3 + 2$

Follow and **label** at least three points.