Section 4.2 – Properties of Rational Functions – Day 1

A **rational function** is a function of the form $R(x)=\dfrac{p(x)}{q(x)}$, where p and q are polynomial functions and q is not the zero polynomial. The domain of a rational function is the set of all real numbers except those for which the denominator q is 0.

Example 1: Find the domain of each of the following rational functions:

a) $f(x)=\dfrac{5x+2}{x+4}$ denom $\neq 0$, so set denom $=0$

$$x+4=0$$
$$x=-4$$

Domain: $\{x \mid x\neq -4\}$

b) $g(x)=\dfrac{3x^4}{x^2+2}$ denom $\neq 0$, so set denom $=0$

$$x^2+2=0$$
$$x^2=-2$$
$$x=\pm\sqrt{-2}\ \Rightarrow x \text{ is not a real number}$$

Domain: All real numbers or $\{x \mid x\in\Re\}$

c) $h(x)=\dfrac{x^2-7}{x-2}$ denom $\neq 0$, so set denom $=0$

$$x-2=0$$
$$x=2$$

Domain: $\{x \mid x\neq 2\}$

If $R(x)=\dfrac{p(x)}{q(x)}$ is a rational function, and if p and q have no common factors, then the rational function R is said to be in **lowest terms**. For a rational function $R(x)=\dfrac{p(x)}{q(x)}$ in lowest terms, the real zeros, if any, of the numerator in the domain of R are the x-intercepts of the graph of R and so will play a major role in the graph of R. The real zeros of the denominator of R, although not in the domain of R, also play a major role in the graph of R.

The domain of a rational function must be found before writing the function in lowest terms.

Consider $f(x)=\dfrac{1}{x^2}$. denom $\neq 0$, so set denom $=0$ Domain: $\{x \mid x\neq 0\}$ $\Rightarrow$ no y-intercept.

$$x^2=0$$
$$x=0$$

There is also no x-intercept since $f(x)=0$ has no solution. $f(x)$ is an even function since $f(-x)=\dfrac{1}{(-x)^2}$

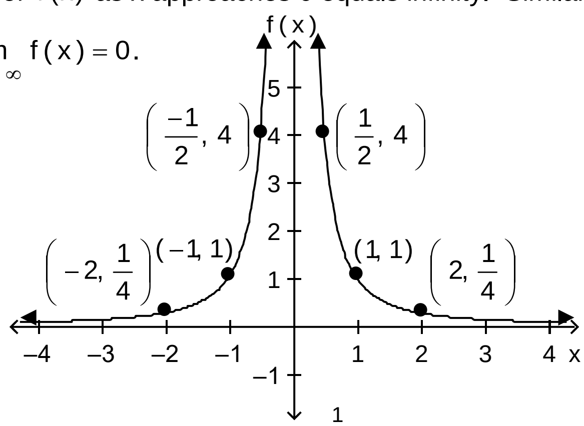
$$\frac{1}{x^2}=0$$
$$1=0$$

False, no solution

$\Rightarrow f(x)$ is symmetric wrt the y-axis

$$=\frac{1}{x^2}$$
$$=f(x)$$

As the values of x approach 0, the values of $f(x)$ become larger and larger positive numbers. It is said that f is unbounded in the positive direction or that f approaches infinity, written $f\to\infty$. In calculus, limits are used, and we say $\lim\limits_{x\to 0} f(x)=\infty$, or the limit of $f(x)$ as x approaches 0 equals infinity. Similarly, as x approaches infinity, the values of $f(x)$ approach 0, i.e., $\lim\limits_{x\to\infty} f(x)=0$.



Section 4.2 – Properties of Rational Functions – Day1 (continued)

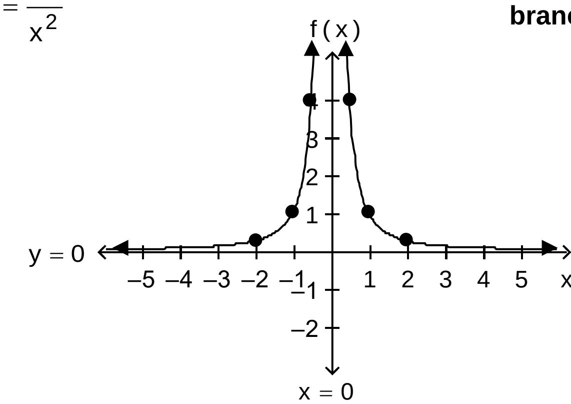
Transformations (shifting, compressing, stretching, and reflecting) can be used to graph rational functions.

Example 2: Graph $h(x) = \dfrac{1}{(x+1)^2} - 2$.

The domain of $h(x)$ is the set of all real numbers except $x = -1$. Domain: $\{x | x -1\}$

State points graphed and Follow three points per branch:

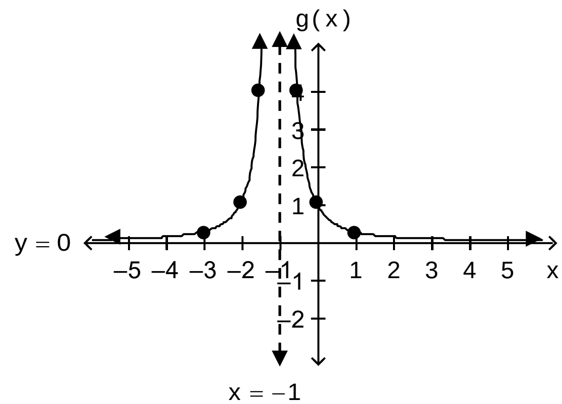
a) Library or Basic Function: $f(x) = \dfrac{1}{x^2}$



x	f(x)

b) Shift left 1 unit: $g(x) = f(x+1)$

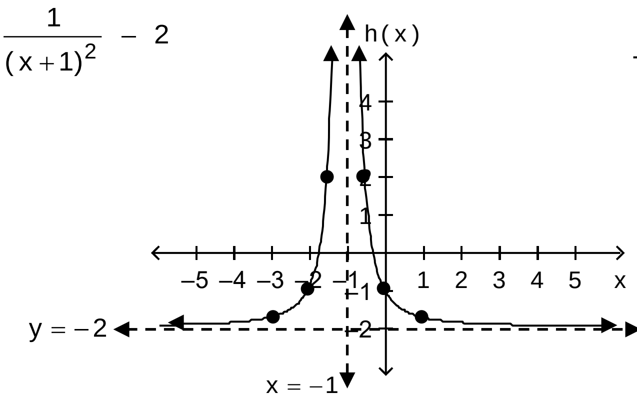
$= \dfrac{1}{(x+1)^2}$



x	g(x)

c) Shift down 2 units: $h(x) = g(x) - 2$

$= \dfrac{1}{(x+1)^2} - 2$



x	h(x)

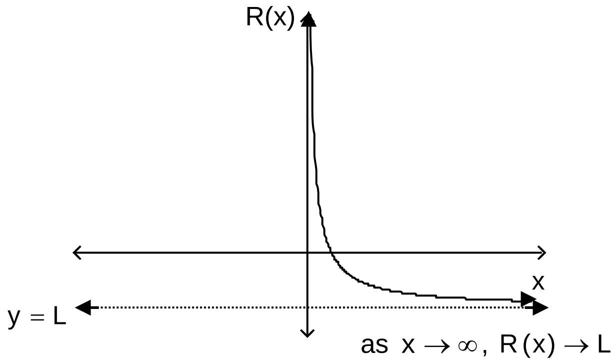
Section 4.2 – Properties of Rational Functions – Day1 (continued)

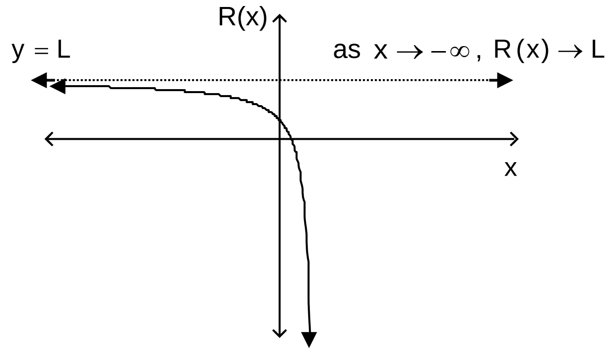
Asymptotes

An asymptote is a line that a certain part of the graph of a function gets closer and closer to but never touches. However, other parts of the graph of the function may intersect a nonvertical asymptote. The graph of a function will never intersect a vertical asymptote.

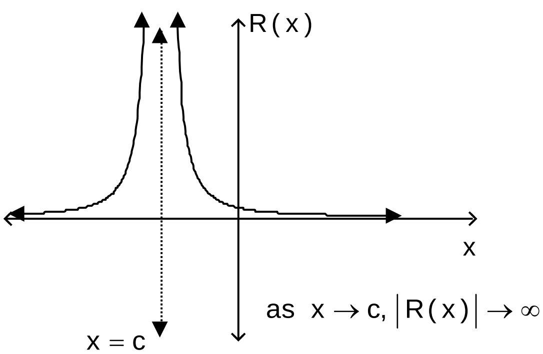
Let R denote a function.

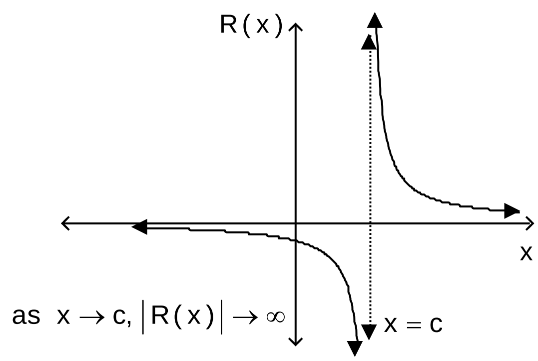
If, as $x \to \infty$ or as $x \to -\infty$, the values of $R(x)$ approach some fixed number, L, then the line $y = L$ is a horizontal asymptote of the graph of R. So, a horizontal asymptote describes a certain behavior of the graph as $x \to \infty$ or as $x \to -\infty$, i.e., its **end behavior**. The graph of a function may intersect a horizontal asymptote (when the absolute value of x is small).



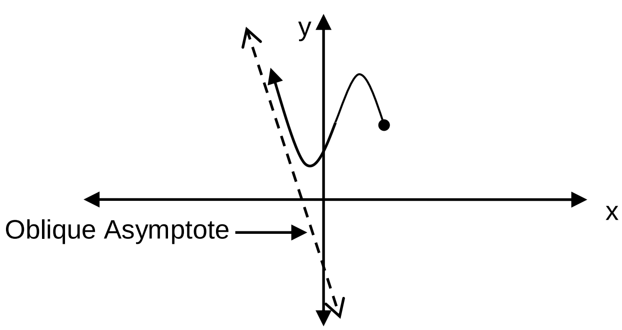


If, as x approaches some number c, the values of $|R(x)| \to \infty$, then the line $x = c$ is a vertical asymptote of the graph of R. So, a vertical asymptote describes a certain behavior of the graph when x is close to some number c. The graph of a rational function will never intersect a vertical asymptote.





An asymptote that is neither horizontal nor vertical is called oblique. An oblique asymptote describes the end behavior of the graph. Oblique asymptotes will be lines of the form $y = ax + b$, i.e., linear equations. The graph of a function may intersect an oblique asymptote.



All material has been taken from *Precalculus,* by M. Sullivan, 10th Edition