Synthetic Division

Long Division

The procedure for dividing two polynomials is similar to the procedure for dividing two integers.

$$(\text{Quotient})(\text{Divisor}) + \text{Remainder} = \text{Dividend}$$

The polynomials must be in standard form.

Example 1: Use Long Division to divide $x^3 + 3x^2 - 4x - 14$ by $x - 2$.

Quotient: $x^2 + 5x + 6$

Divisor: $x - 2$ Dividend: $x^3 + 3x^2 - 4x - 14$

$$\begin{array}{r} x^2 + 5x + 6 \\ x - 2 \overline{)\, x^3 + 3x^2 - 4x - 14} \\ \underline{-(x^3 - 2x^2)} \\ 5x^2 - 4x \\ \underline{-(5x^2 - 10x)} \\ 6x - 14 \\ \underline{-(6x - 12)} \\ -2 \end{array}$$

Remainder: -2

Check:

$$\begin{aligned} (\text{Quotient})(\text{Divisor}) + \text{Remainder} &= (x^2 + 5x + 6)(x - 2) + -2 \\ &= (x^3 + 5x^2 + 6x) - (2x^2 + 10x + 12) - 2 \\ &= x^3 + 5x^2 + 6x - 2x^2 - 10x - 12 - 2 \\ &= x^3 + 3x^2 - 4x - 14 \\ &= \text{Dividend} \end{aligned}$$

Thus, $$\frac{x^3 + 3x^2 - 4x - 14}{x - 2} = x^2 + 5x + 6 - \frac{2}{x - 2}$$

The process of dividing polynomials leads to the following theorem:

Theorem: Let Q be a polynomial of positive degree and let P be a polynomial whose degree is greater than the degree of Q. The remainder after dividing P by Q is either the zero polynomial or a polynomial whose degree is less than the degree of the divisor Q.

Synthetic Division

You may use long division to divide polynomial functions. You can also use a shortened form of long division called synthetic division.

The process of synthetic division involves rewriting long division in a more compact form, using simpler notation. In synthetic division, you do not write the variables.

Synthetic division can be used to divide a polynomial only by a linear binomial of the form $x - r$. When dividing by nonlinear divisors, use long division.

Synthetic Division (continued)

Example 2: Use synthetic division to divide $x^3+3x^2-4x-14$ by $x-2$.

Synthetic Division:

$$\begin{array}{r|rrrr} 2 & 1 & 3 & -4 & -14 \\ & \downarrow & & & \\ \hline & 1 & & & \end{array}$$

Write the r value of the divisor $(x-r)$, then write the coefficients of the polynomial (include zero place holders). Write the first coefficient below the line.

$$\begin{array}{r|rrrr} 2 & 1 & 3 & -4 & -14 \\ \bullet & & 2 & & \\ \hline & 1 & 5 & & \end{array}$$

Multiply the r value, 2, by the number below the line, and write the product below the next coefficient. Write the sum of 3 and 2 below the line.

$$\begin{array}{r|rrrr} 2 & 1 & 3 & -4 & -14 \\ \bullet & & 2 & 10 & \\ \hline & 1 & 5 & 6 & \end{array}$$

Multiply the r value, 2, by the number below the line, and write the product below the next coefficient. Write the sum of -4 and 10 below the line.

$$\begin{array}{r|rrrr} 2 & 1 & 3 & -4 & -14 \\ \bullet & & 2 & 10 & 12 \\ \hline & 1 & 5 & 6 & -2 \end{array}$$

Multiply the r value, 2, by the number below the line, and write the product below the next coefficient. Write the sum of -14 and 12 below the line. The last remaining number, -2, is the remainder.

Check:

$$\begin{aligned}(\text{Quotient})(\text{Divisor}) + \text{Remainder} &= (x^2+5x+6)(x-2) + -2 \\ &= (x^3+5x^2+6x) - (2x^2+10x+12) - 2 \\ &= x^3+5x^2+6x - 2x^2 - 10x - 12 - 2 \\ &= x^3+3x^2-4x-14 \\ &= \text{Dividend}\end{aligned}$$

Thus, $$\frac{x^3+3x^2-4x-14}{x-2} = x^2+5x+6-\frac{2}{x-2}$$

Remember, the final entry in the row is the remainder. The other entries are the coefficients, in descending order, of a polynomial (the Quotient) whose degree is one less than that of the dividend.

$(\text{Quotient})(\text{Divisor}) + \text{Remainder} = \text{Dividend}$

The divisor must be of the form $x-r$, so if you have $x+r$, rewrite it as $x-(-r)$.

Also, when using synthetic division, remember to put in placeholders of 0 for powers of x with 0 coefficients. For example, if you want to divide x^3+2x-6 by $x-3$, then the synthetic division set-up would look like

$3\overline{)\;1 \quad 0 \quad 2 \quad -6}$, where 0 is the placeholder for the missing x^2 term.