

Section 5.4 – Logarithmic Functions – Day 2Solve Logarithmic Equations

Equations that contain logarithms are called **logarithmic equations**. Care must be taken when solving logarithmic equations algebraically. In the expression  $\log_a M$ , remember that  $a$  and  $M$  are positive and  $a \neq 1$ . Be sure to check each apparent solution in the original equation and discard any that are extraneous.

Some logarithmic equations can be solved by changing the logarithmic equation to exponential form using the fact that  $y = \log_a x$  means  $a^y = x$ .

Example 8: Solve:  $\log_4(3x - 8) = 2$ .

To solve, change the logarithmic equation to exponential form.

$$\log_4(3x - 8) = 2$$

$$3x - 8 = 4^2 \quad \text{Change to exponential form}$$

$$3x - 8 = 16$$

$$3x = 24$$

$$x = 8$$

$$\begin{aligned} \text{Check: If } x = 8, \log_4(3x - 8) &= \log_4(3(8) - 8) \\ &= \log_4(24 - 8) \\ &= \log_4 16 \\ &= 2 \quad \text{since } 4^2 = 16 \end{aligned}$$

Example 9: Solve:  $\log_x(243) = 5$ .

To solve, change the logarithmic equation to exponential form.

$$\log_x(243) = 5$$

$$x^5 = 243 \quad \text{Change to exponential form}$$

$$x^5 = 3^5$$

$$\Rightarrow x = 3$$

$$\begin{aligned} \text{Check: If } x = 3, \log_x(243) = 5 \text{ becomes } \log_3(243) &= 5 \\ \Rightarrow 3^5 &= 243 \\ 243 &= 243 \end{aligned}$$

Example 10: Solve:  $e^{4x} = 6$ .

To solve, change the exponential equation to logarithmic form.

$$e^{4x} = 6$$

$$\ln(6) = 4x \quad \text{Change to logarithmic form}$$

$$x = \frac{\ln(6)}{4}$$

$$x \approx 0.4479398$$

$$x \approx 0.447940$$

$$\begin{aligned} \text{Check: If } x = 0.447940, e^{4x} &= e^{4(0.447940)} \\ &\approx e^{1.79176} \\ &\approx 6.000003 \\ &\approx 6 \end{aligned}$$