Section 5.5 – Properties of Logarithms – Day 2

Evaluate Logarithms Whose Base is Neither 10 Nor e

Logarithms with base 10 – common logarithms – were used to facilitate arithmetic computations before the widespread use of calculators. Natural logarithms – that is, logarithms whose base is the number e – remain very important because they arise frequently in the study of natural phenomena.

Common logarithms are usually abbreviated by writing **log**, with the base understood to be 10, just as the natural logarithms are abbreviated **ln**, with the base understood to be *e*.

Most calculators have both [log] and [ln] keys to calculate the common logarithm and the natural logarithm of a number, respectively. But what if the base is neither 10 nor *e*?

Example 5: Evaluate $\log_3 5$.

Set $y = \log_3 5$

$\Rightarrow 3^y = 5$ change to exponential form

Take the $\ln$ of both sides:

$$\ln\left(3^y\right) = \ln 5$$
$$y \ln 3 = \ln 5$$
$$y = \frac{\ln 5}{\ln 3}$$
$$y \approx 1.464974$$
$$y \approx 1.46$$

The Change-of-Base Formula allows you to change from base a to base b.

Theorem: Change-of-Base Formula

If $a \neq 1$, $b \neq 1$, and M are positive real numbers, then $\log_a M = \dfrac{\log_b M}{\log_b a}$.

Since your calculator has $\log_{10} \equiv \log$ and $\log_e \equiv \ln$ keys, then $\log_a M = \dfrac{\log M}{\log a}$ or $\log_a M = \dfrac{\ln M}{\ln a}$.

Example 6: Evaluate $\log_4(20)$.

$$\log_4(20) = \frac{\log(20)}{\log 4} \approx \frac{1.301030}{0.602060} \approx 2.160964 \approx 2.16$$

OR

$$\log_4(20) = \frac{\ln(20)}{\ln 4} \approx \frac{2.995732}{1.386294} \approx 2.160964 \approx 2.16$$

To graph logarithmic functions when the base is different from 10 or e requires the Change-of-Base Formula. For example, to graph $\log_4 x$, graph $\dfrac{\log x}{\log 4}$ or $\dfrac{\ln x}{\ln 4}$.

All material has been taken from *Precalculus,* by M. Sullivan, 10th Edition