Section 6.4 – Graphs of the Sine and Cosine Functions – Day 1

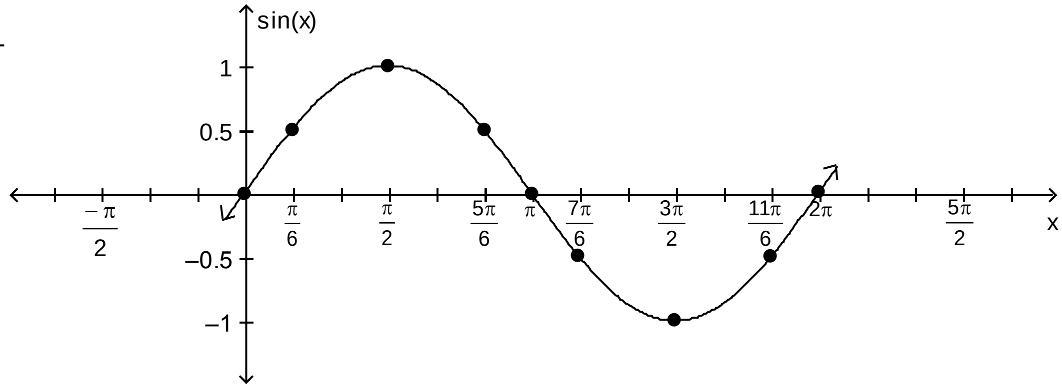
In this section, we will use radian measure for the independent variable x.

Label the five (5) key points on each graph.

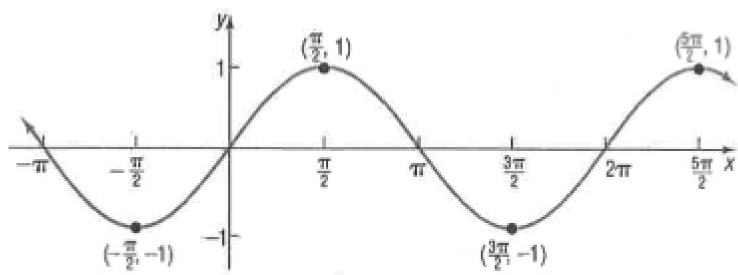
The Graph of $f(x) = \sin x$

The sine function has period 2π, so we need to graph $f(x) = \sin x$ on the interval $[0, 2\pi]$. The remainder of the graph will consist of repetitions of this portion of the graph.

x	$f(x) = \sin x$
0	0
$\pi/6$	$1/2$
$\pi/2$	1
$5\pi/6$	$1/2$
π	0
$7\pi/6$	$-1/2$
$3\pi/2$	-1
$11\pi/6$	$-1/2$
2π	0



This is one period, or **cycle**, of the graph of $f(x) = \sin x$. To obtain a more complete graph, repeat this period in each direction, as shown in the graph below.



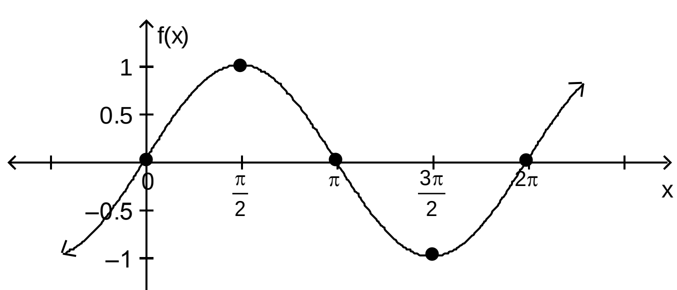
Characteristics of the Sine Function:

1) The domain is the set of all real numbers.
2) The range consists of all real numbers from -1 to 1, inclusive.
3) The sine function is an odd function $\Rightarrow$ Symmetric with respect to the origin.
4) The sine function is periodic, with period 2π.
5) The x-intercepts are $\ldots, -2\pi, -\pi, 0, \pi, 2\pi, 3\pi, \ldots,$ i.e., integral multiples of π.
   The y-intercept is 0.
6) The maximum value is 1 and occurs at $x = \ldots, \frac{-3\pi}{2}, \frac{\pi}{2}, \frac{5\pi}{2}, \ldots$.
   The minimum value is -1 and occurs at $x = \ldots, \frac{-\pi}{2}, \frac{3\pi}{2}, \frac{7\pi}{2}, \ldots$.

The transformations from chapter 2 (shifting, compressing, stretching, and reflecting) are used to graph variations of the trigonometric functions.

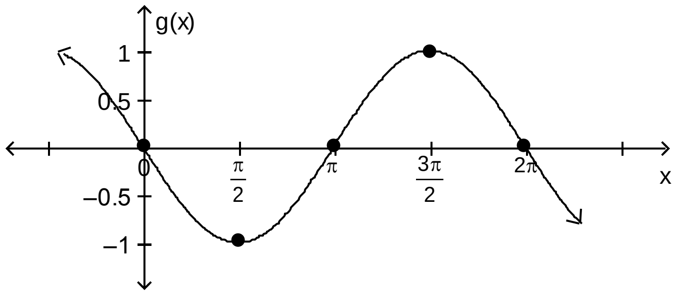
Example 1: Use the graph of $f(x) = \sin x$ to graph $h(x) = -\sin x + 4$

a) $f(x) = \sin x$, Library/Basic function

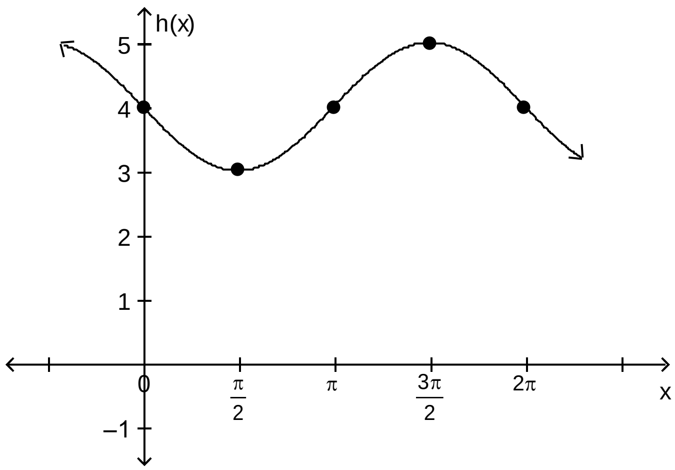


Section 6.4 – Graphs of the Trigonometric Functions – Day 1 (continued)

b) $g(x) = -f(x)$, reflect about the x-axis
$= -\sin x$



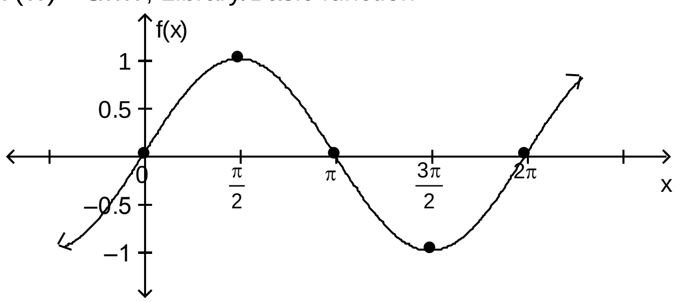
c) $h(x) = g(x) + 4$, shift up 4 units
$= -\sin x + 4$



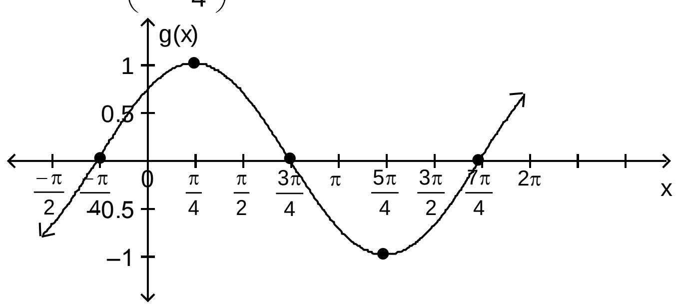
The domain of $h(x) = -\sin x + 4$ is $(-\infty, \infty)$. The range is $[3,5]$.

Example 2: Use the graph of $f(x) = \sin x$ to graph $g(x) = \sin\left(x + \frac{\pi}{4}\right)$

a) $f(x) = \sin x$, Library/Basic function



b) $g(x) = f\left(x + \frac{\pi}{4}\right)$, shift left $\frac{\pi}{4}$ units
$= \sin\left(x + \frac{\pi}{4}\right)$



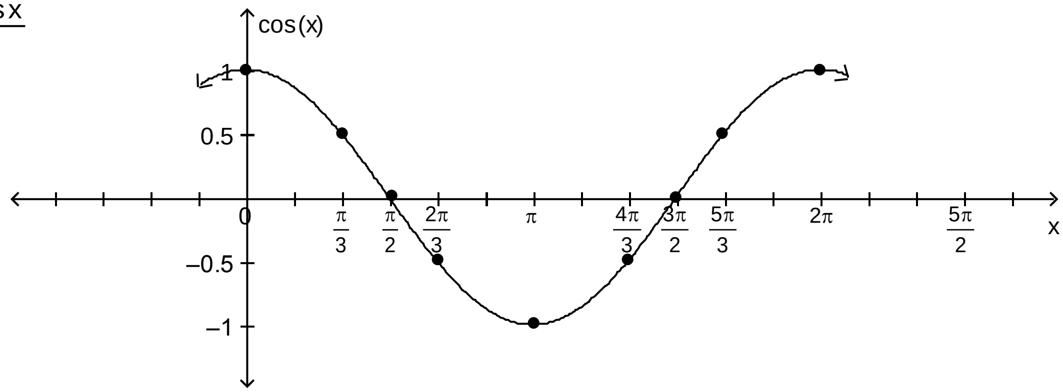
The domain of $g(x) = \sin\left(x + \frac{\pi}{4}\right)$ is $(-\infty, \infty)$.
The range is $[-1,1]$.

Section 6.4 – Graphs of the Trigonometric Functions – Day 1 (continued)

The Graph of $f(x) = \cos x$

The cosine function also has period 2π, so we need to graph $f(x) = \cos x$ on the interval $[0, 2\pi]$. The remainder of the graph will consist of repetitions of this portion of the graph.

x	$f(x) = \cos x$
0	1
$\pi/3$	$1/2$
$\pi/2$	0
$2\pi/3$	$-1/2$
π	-1
$4\pi/3$	$-1/2$
$3\pi/2$	0
$5\pi/3$	$1/2$
2π	1



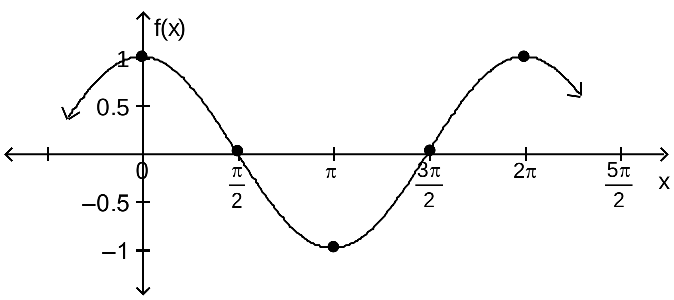
This is one period, or cycle, of the graph of $f(x) = \cos x$. To obtain a more complete graph, repeat this period in each direction.

Characteristics of the Cosine Function:

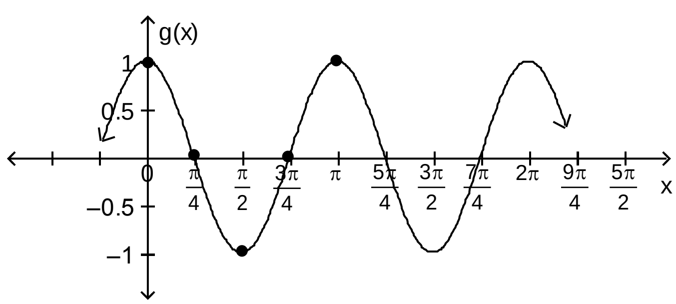
1) The domain is the set of all real numbers.
2) The range consists of all real numbers from -1 to 1, inclusive.
3) The cosine function is an even function $\Rightarrow$ Symmetric with respect to the y-axis.
4) The cosine function is periodic, with period 2π.
5) The x-intercepts are $\ldots, -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$.
   The y-intercept is 1.
6) The maximum value is 1 and occurs at $x = \ldots, -2\pi, 0, 2\pi, 4\pi, 6\pi, \ldots$.
   The minimum value is -1 and occurs at $x = \ldots, -\pi, \pi, 3\pi, 5\pi, \ldots$.

Example 3: Use the graph of $f(x) = \cos x$ to graph $g(x) = \cos(2x)$.

a) $f(x) = \cos x$, Library/Basic function



b) $g(x) = f(2x)$
$= \cos(2x)$
horizontal compression by a factor of $\frac{1}{2}$



The domain of $g(x)$ is $(-\infty, \infty)$ and the range is $[-1, 1]$.