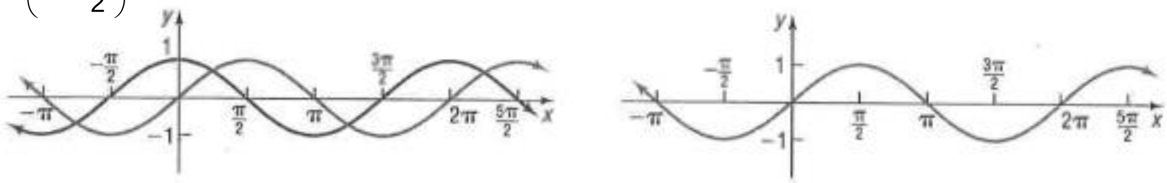
**Label the five (5) key points on each graph in Examples 2, 3, & 4.**

## Determine the Amplitude and Period of Sinusoidal Functions

After a horizontal shift of $\frac{\pi}{2}$ units to the right, the graph of $y = \cos x$ coincides with the graph of $y = \sin x$, i.e., $\sin x = \cos\left(x - \frac{\pi}{2}\right)$. See the graph below.

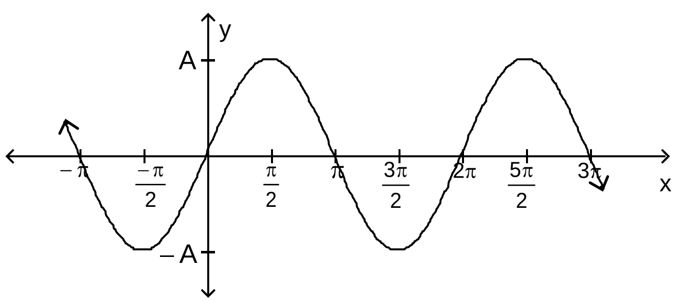



(a) $y = \cos x$ $y = \cos\left(x - \frac{\pi}{2}\right)$ (b) $y = \sin x$

Because of this relationship, the graphs of functions of the form $y = A\sin(\omega x)$ or $y = A\cos(\omega x)$ are referred to as **sinusoidal graphs**.

The values of the functions $y = A\sin x$ and $y = A\cos x$, where $A \neq 0$, will always satisfy the inequalities $-|A| \leq A\sin x \leq |A|$ and $-|A| \leq A\cos x \leq |A|$. The number $|A|$ is called the **amplitude** of $y = A\sin x$ or $y = A\cos x$,

Example 1: Let $y = A\sin x,\quad A > 0$. Then the period is $2\pi$ and the graph looks like




One period of the graph of $y = \sin(\omega x)$ or $y = \cos(\omega x)$ is called a **cycle**. $\omega$ is the number of cycles per $2\pi$.

If $\omega > 0$, the functions $y = \sin(\omega x)$ and $y = \cos(\omega x)$ will have period $\frac{2\pi}{\omega}$.

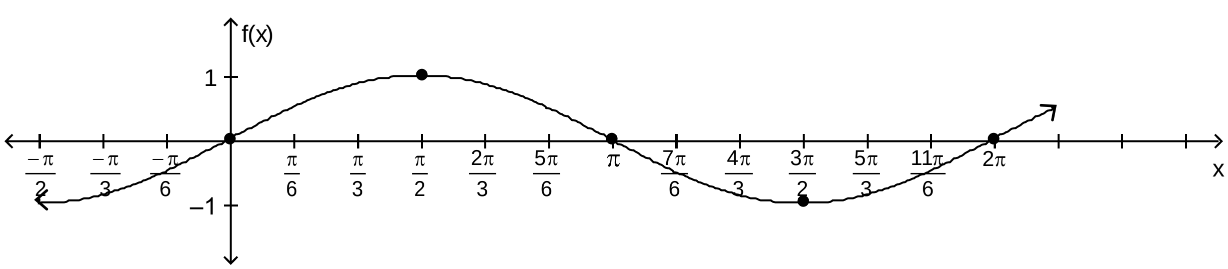
If $\omega < 0$, use the Even-Odd Properties to get an equivalent form in which the coefficient of x is positive. In other words, you want to **work with a positive $\omega$**, so if it is not, use the Even-Odd properties to get a positive $\omega$.

sin is odd, so $\sin(\omega x) = -\sin(-\omega x)$ or cos is even, so $\cos(\omega x) = \cos(-\omega x)$

Example 2: Graph $g(x) = \sin(3x)$.

$$g(x) = \sin(\omega x)$$
$$= \sin(3x) \Rightarrow \omega = 3 \text{ and the period is } \frac{2\pi}{\omega} = \frac{2\pi}{3}.$$

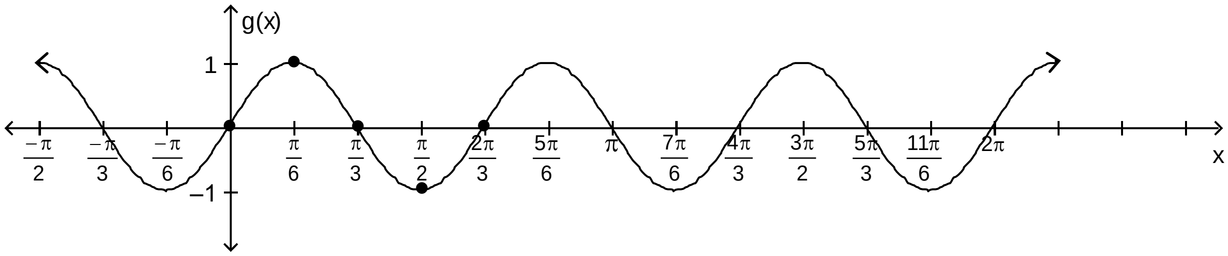
a) Let $f(x) = \sin x$, the Library/Basic function.



Section 6.4 – Graphs of the Sine and Cosine Functions – Day 2 (continued)

b) $g(x) = f(3x)$

$= \sin(3x)$, a horizontal compression of $f(x)$ by a factor of $\frac{1}{3}$.




**Theorem**: If $\omega > 0$, the amplitude and period of $y = A\sin(\omega x)$ and $y = A\cos(\omega x)$ are given by

Amplitude = $|A|$ and Period = T

$= \frac{2\pi}{\omega}$

## Graph Sinusoidal Functions Using Key Points

We have been using transformations to graph functions of the form $y = A\sin(\omega x)$ or $y = A\cos(\omega x)$. The following shows you another method that can be used to graph these functions.

When graphing a sinusoidal function of the form $y = A\sin(\omega x)$ or $y = A\cos(\omega x)$, use the amplitude to determine the maximum and minimum values of the function ( i.e., to scale the y-axis). The period is used to divide the x-axis into four subintervals. The endpoints of the subintervals give rise to five key points on the graph, which are used to sketch one cycle. Extend the graph in either direction to make it complete.

Example 3: Determine the amplitude and period of $y = 2\cos(3x)$, and graph the function.

$y = A\cos(\omega x)$

$= 2\cos(3x)$

$\Rightarrow$ $A = 2$, $\omega = 3$. A is positive, so the graph is a non-reflected cosine function.

Amplitude = $|A|$ and Period = T

$= |2|$ $\qquad$ $= \frac{2\pi}{\omega}$

$= 2$ $\qquad$ $= \frac{2\pi}{3}$

The graph of $y = 2\cos(3x)$ will lie between $-2$ and 2 on the y-axis. One cycle will begin at $x = 0$ and end at $x = \frac{2\pi}{3}$. Divide the interval $\left[0, \frac{2\pi}{3}\right]$ into four subintervals, each of length

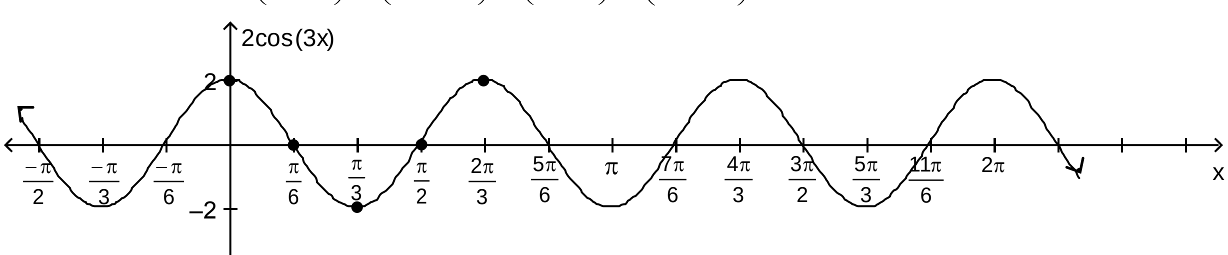
Five key points Method

$\frac{2\pi}{3} \div 4 = \frac{\pi}{6}$: $\left[0, \frac{\pi}{6}\right], \left[\frac{\pi}{6}, \frac{2\pi}{6}\right], \left[\frac{2\pi}{6}, \frac{3\pi}{6}\right], \left[\frac{3\pi}{6}, \frac{4\pi}{6}\right]$

$= \left[0, \frac{\pi}{6}\right], \left[\frac{\pi}{6}, \frac{\pi}{3}\right], \left[\frac{\pi}{3}, \frac{\pi}{2}\right], \left[\frac{\pi}{2}, \frac{2\pi}{3}\right]$

The endpoints of these intervals give rise to five key points on the graph:

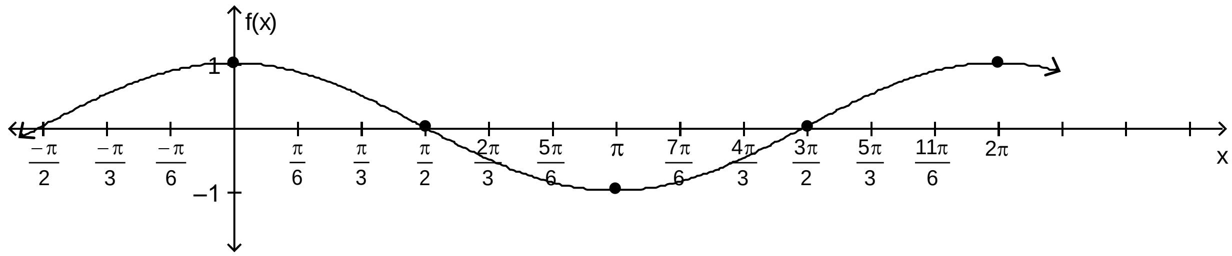
$(0, 2)$, $\left(\frac{\pi}{6}, 0\right)$, $\left(\frac{\pi}{3}, -2\right)$, $\left(\frac{\pi}{2}, 0\right)$, $\left(\frac{2\pi}{3}, 2\right)$



Section 6.4 – Graphs of the Sine and Cosine Functions – Day 2 (continued)

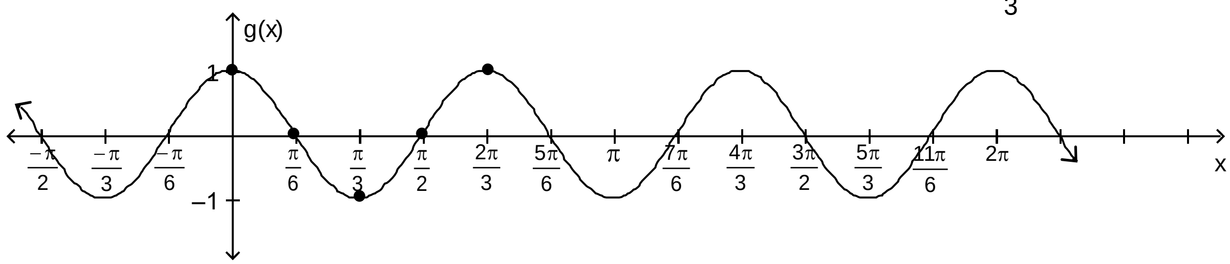
**OR** Use transformations to graph $h(x) = 2\cos(3x)$

a) Let $f(x) = \cos x$, the Library/Basic function.



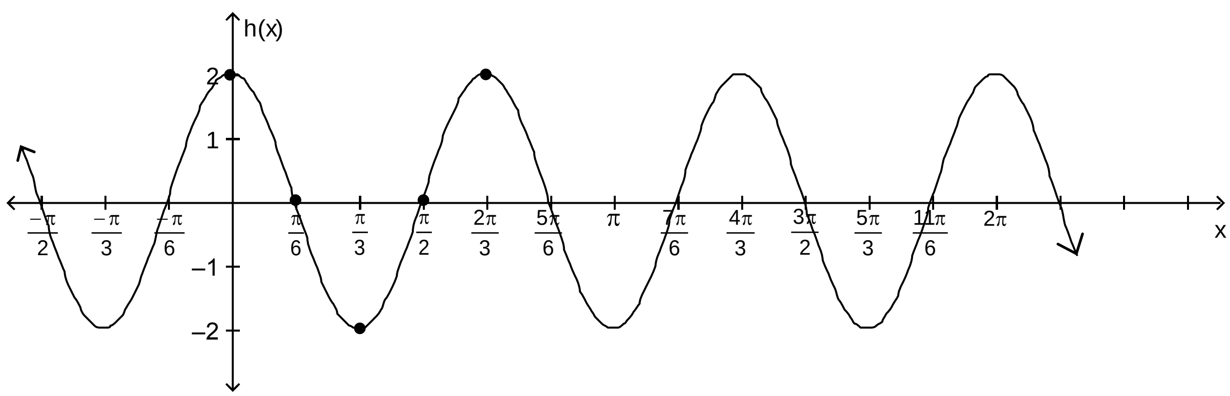

b) $g(x) = f(3x)$

$= \cos(3x)$, a horizontal compression of $f(x)$ by a factor of $\frac{1}{3}$




c) $h(x) = 2g(x)$

$= 2\cos(3x)$, a vertical stretch of $g(x)$ by a factor of 2




**Steps for Graphing a Sinusoidal Function of the Form $y = A\sin(\omega x)$ or $y = A\cos(\omega x)$ Using Key Points**

**Step1**: Determine the amplitude and period of the sinusoidal function.

**Step 2**: Divide the interval $\left[0, \frac{2\pi}{\omega}\right]$ into four subintervals of the same length.

**Step 3**: Use the endpoints of these subintervals to obtain five key points on the graph.

**Step 4**: Plot the five key points, and draw a sinusoidal graph to obtain the graph of one cycle. Extend the graph in each direction to make it complete.

## Section 6.4 – Graphs of the Sine and Cosine Functions – Day 2 (continued)

Example 4: Determine the amplitude and period of $y = -3\sin(\pi x)$, and graph the function.

$y = A\sin(\omega x)$
$= -3\sin(\pi x)$

$\Rightarrow$ $A = -3$, $\omega = \pi$. A is negative, so the graph is a reflected sine function.

Amplitude = $|A|$ and Period = T

$= |-3|$ $\quad$ $= \dfrac{2\pi}{\omega}$

$= 3$ $\quad$ $= \dfrac{2\pi}{\pi}$

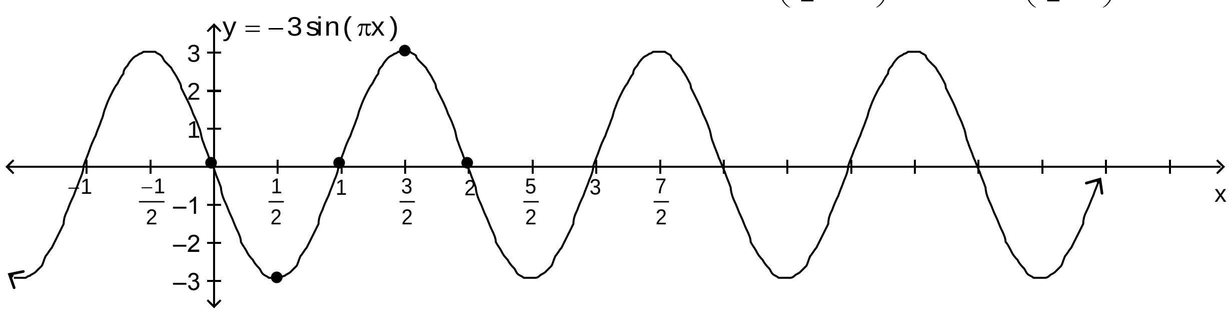
$= 2$

Graph $y = -3\sin(\pi x)$.

Five key points Method

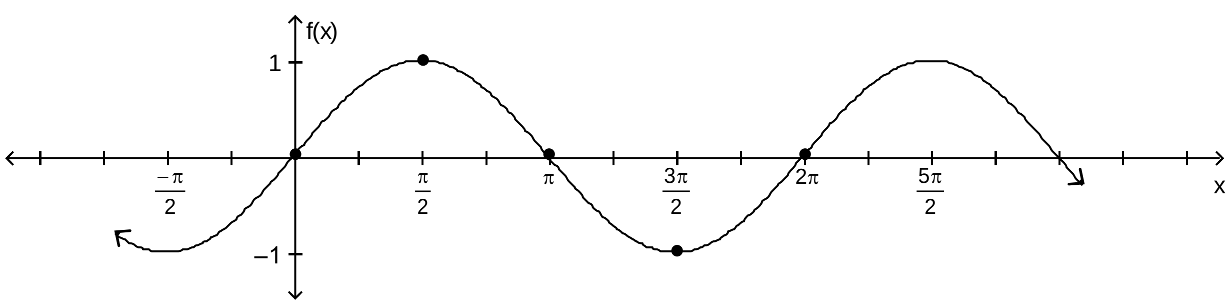
The graph of $y = -3\sin(\pi x)$ will lie between $-3$ and 3 on the y-axis. One cycle will begin at $x = 0$ and end at $x = 2$. Divide the interval $[0, 2]$ into four subintervals, each of length $2 \div 4 = \frac{1}{2}$: $\left[0, \frac{1}{2}\right], \left[\frac{1}{2}, \frac{2}{2}\right], \left[\frac{2}{2}, \frac{3}{2}\right], \left[\frac{3}{2}, \frac{4}{2}\right] = \left[0, \frac{1}{2}\right], \left[\frac{1}{2}, 1\right], \left[1, \frac{3}{2}\right], \left[\frac{3}{2}, 2\right]$

The endpoints of these intervals give rise to five key points on the graph: $(0, 0)$, $\left(\frac{1}{2}, -3\right)$, $(1, 0)$, $\left(\frac{3}{2}, 3\right)$, $(2, 0)$



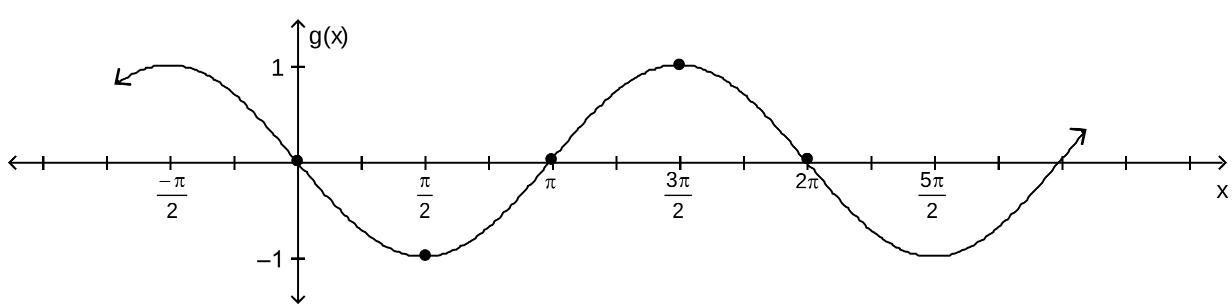

**OR** Use transformations to graph $k(x) = -3\sin(\pi x)$.

a) Let $f(x) = \sin x$, the Library/Basic function.




b) $g(x) = -f(x)$

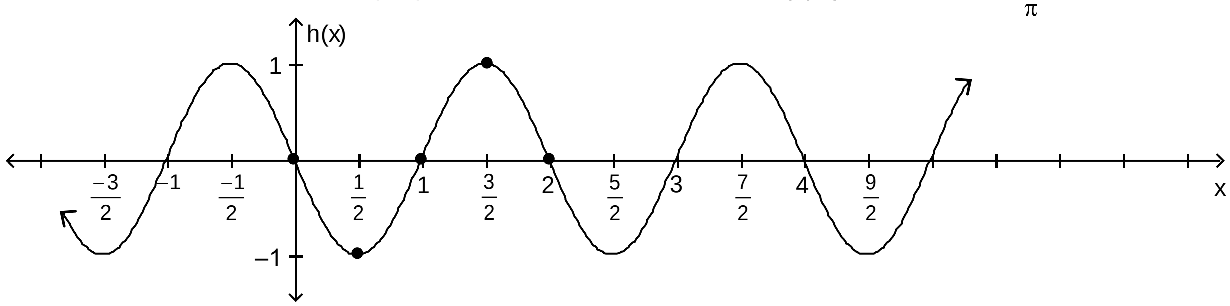
$= -\sin x$, a reflection of $f(x)$ about the x-axis



Section 6.4 – Graphs of the Sine and Cosine Functions – Day 2 (continued)

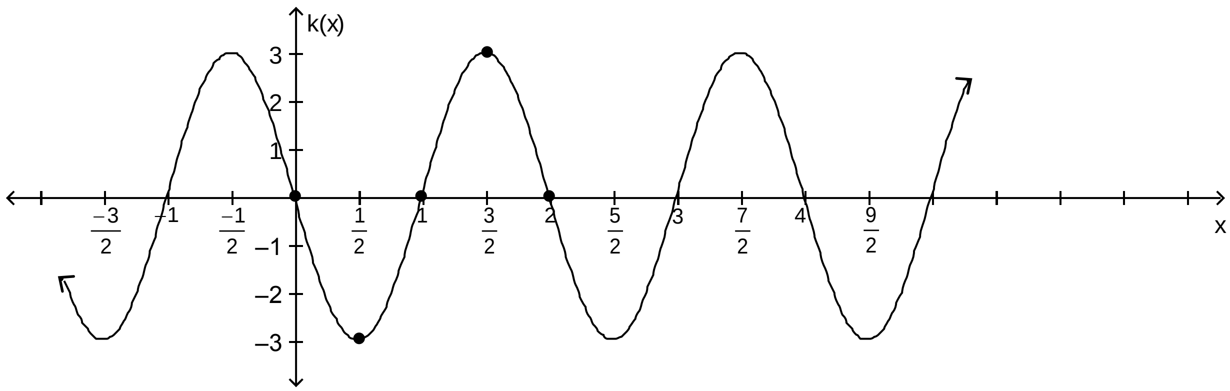
c) $h(x) = g(\pi x)$

$= -\sin(\pi x)$, a horizontal compression of $g(x)$ by a factor of $\frac{1}{\pi}$




d) $k(x) = 3h(x)$

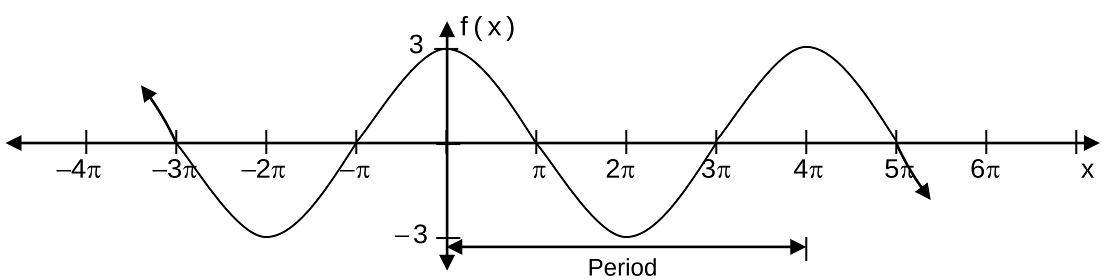
$= -3\sin(\pi x)$, a vertical stretch of $h(x)$ by a factor of 3




## Find an equation for a Sinusoidal Graph

You can also use amplitude and period to identify a sinusoidal function when its graph is given.

Example 5: Find an equation for the graph shown below.




This is the graph of a non-reflected cosine function with amplitude $|A| = 3$ and period $T = 4\pi$. Since the graph has a positive value at $x = 0$, it looks like a basic cosine function with $A = 3$.
(You could also consider this to be a sine function with a horizontal shift, but the cosine function is easier.)

Period $T = 4\pi \quad \Rightarrow \quad \frac{2\pi}{\omega} = 4\pi$

$$2\pi = 4\pi\omega$$

$$\frac{2\pi}{4\pi} = \omega$$

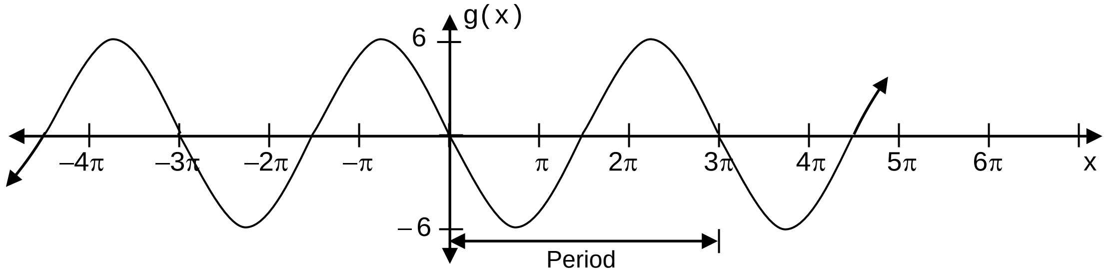
$$\omega = \frac{1}{2}$$

A cosine function for the graph above is $f(x) = A\cos(\omega x)$

$$\Rightarrow f(x) = 3\cos\left(\frac{1}{2}x\right)$$

## Section 6.4 – Graphs of the Sine and Cosine Functions – Day 2 (continued)

Example 6: Find an equation for the graph shown below.




This is the graph of a reflected sine function with amplitude $|A| = 6$ and period $T = 3\pi$.

Since the graph goes through the origin and it is decreasing near the origin, it looks like a basic sine function that has been reflected about the x-axis, so $A = -6$.

(You could also consider this to be a cosine function with a horizontal shift, but the sine function is easier.)

Period $T = 3\pi \quad \Rightarrow \quad \frac{2\pi}{\omega} = 3\pi$

$$2\pi = 3\pi\omega$$

$$\frac{2\pi}{3\pi} = \omega$$

$$\omega = \frac{2}{3}$$

A sine function for the graph above is $g(x) = A\sin(\omega x)$

$$\Rightarrow \; g(x) = -6\sin\left(\frac{2}{3}x\right)$$