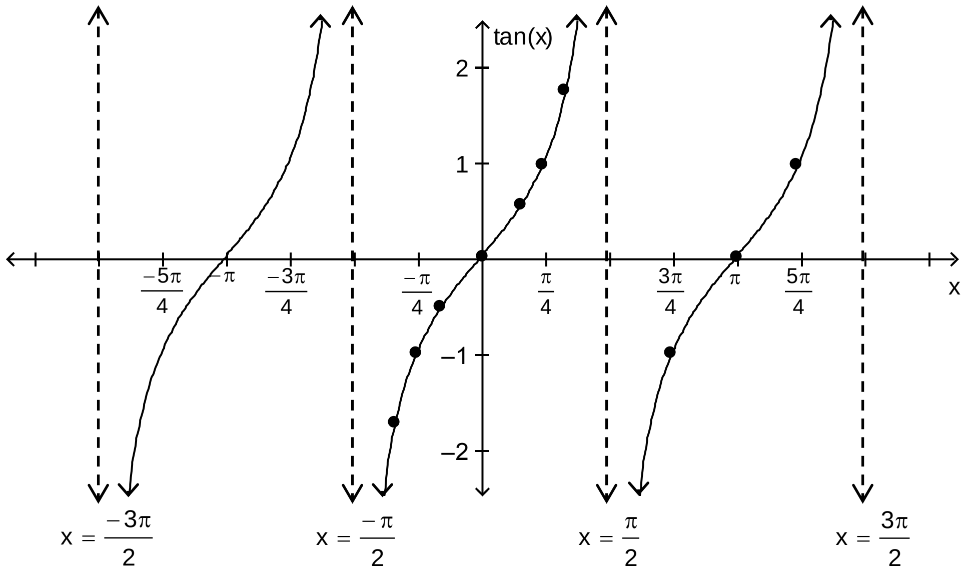
Section 6.5 – Graphs of the Tangent, Cotangent, Secant, and Cosecant Functions

Label three (3) key points on the two highlighted branches of each graph.

The Graph of $f(x) = \tan x$

The tangent function has period π. The tangent is not defined at $\ldots, \frac{-3\pi}{2}, \frac{-\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \ldots,$ so we will look at the interval $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$. The rest of the graph will consist of repetitions of this portion of the graph.

x	$f(x) = \tan x$
$-\pi/2$	undefined
$-\pi/3$	$-\sqrt{3} \approx -1.73$
$-\pi/4$	-1
$-\pi/6$	$-\sqrt{3}/3 \approx -0.58$
0	0
$\pi/6$	$\sqrt{3}/3 \approx 0.58$
$\pi/4$	1
$\pi/3$	$\sqrt{3} \approx 1.73$
$\pi/2$	undefined



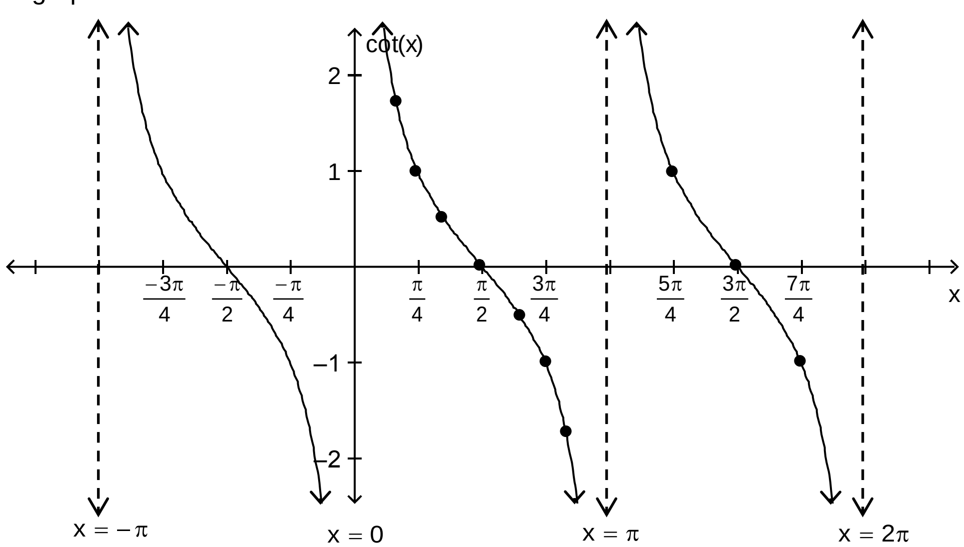
Characteristics of the Tangent Function:

1) The domain is the set of all real numbers, except odd multiples of $\frac{\pi}{2}$.
2) The range consists of all real numbers.
3) The tangent function is an odd function $\Rightarrow$ Symmetric with respect to the origin.
4) The tangent function is periodic, with period π.
5) The x-intercepts are $\ldots, -2\pi, -\pi, 0, \pi, 2\pi, 3\pi, \ldots,$ integral multiples of π.
   The y-intercept is 0.
6) Vertical asymptotes occur at $x = \ldots, \frac{-3\pi}{2}, \frac{-\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \ldots$.

The Graph of $f(x) = \cot x$.

The cotangent function has period π. The cotangent is not defined at integral multiples of π $(\ldots, -2\pi, -\pi, 0, \pi, 2\pi, \ldots)$, so we will look at the interval $(0, \pi)$. The rest of the graph will consist of repetitions of this portion of the graph.

x	$f(x) = \cot x$
0	undefined
$\pi/6$	$\sqrt{3} \approx 1.73$
$\pi/4$	1
$\pi/3$	$\sqrt{3}/3 \approx 0.58$
$\pi/2$	0
$2\pi/3$	$-\sqrt{3}/3 \approx -0.58$
$3\pi/4$	-1
$5\pi/6$	$-\sqrt{3} \approx -1.73$
π	undefined



Section 6.5 – Graphs of the Tangent, Cotangent, Secant, and Cosecant Functions (continued)

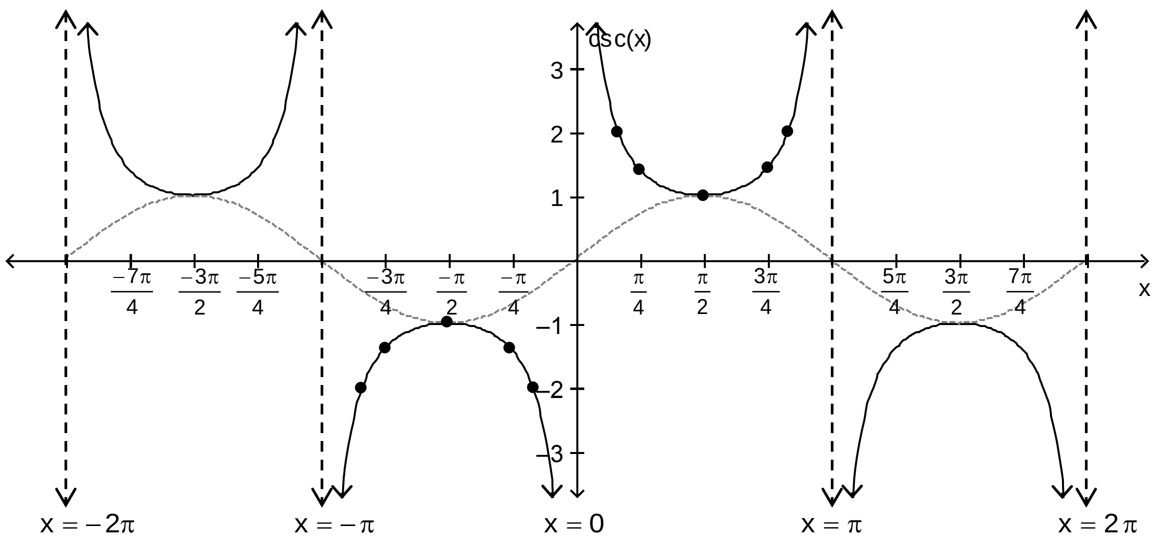
For tangent and cotangent functions, there is no concept of amplitude since the range of the tangent and cotangent functions is $(-\infty,\infty)$. The role of A in $y = A\tan(\omega x)+B$ and $y = A\cot(\omega x)+B$ is to provide the magnitude of the vertical stretch. The period of $y=\tan x$ and $y = \cot x$ is π, so the period of $y = A\tan(\omega x)+B$ and $y = A\cot(\omega x)+B$ is $\frac{\pi}{\omega}$, caused by the horizontal compression of the graph by a factor of $\frac{1}{\omega}$. Finally, the presence of B indicates that a vertical shift is required.

The Graphs of $f(x)=\csc x$, and $f(x)=\sec x$.

The cosecant and secant functions, referred to as reciprocal functions, are graphed using the reciprocal identities $\csc x = \frac{1}{\sin x}$ and $\sec x = \frac{1}{\cos x}$.

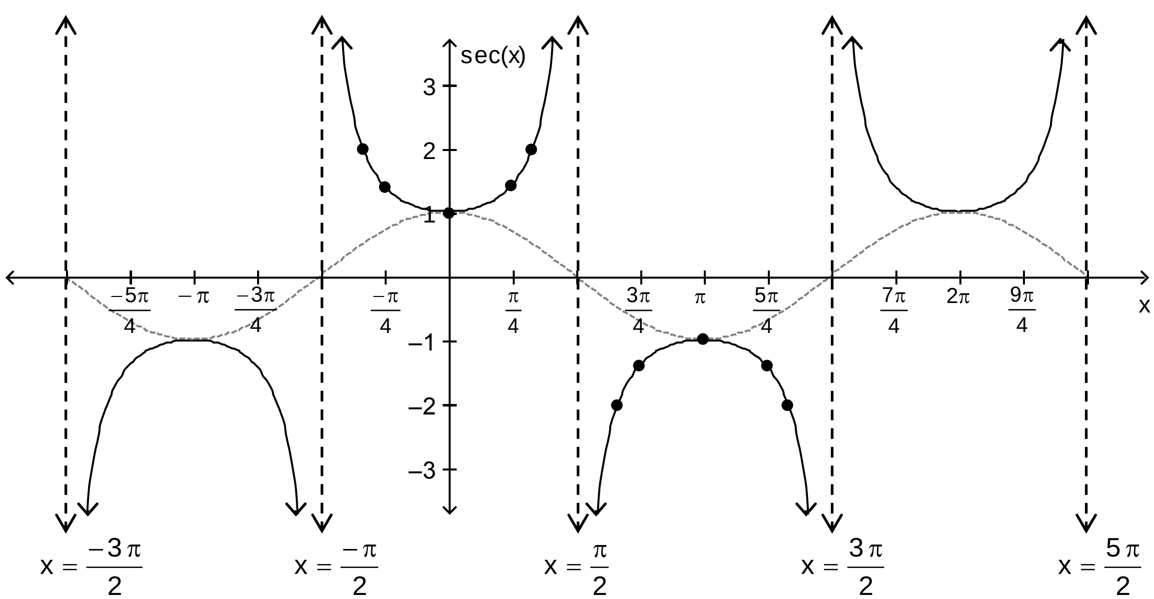
Since the sine function is 0 at integral multiples of π, the cosecant function has vertical asymptotes at integral multiples of π.

x	f(x)
$-5\pi/6$	-2
$-3\pi/4$	-1.41
$-\pi/2$	-1
$-\pi/4$	-1.41
$-\pi/6$	-2
$\pi/6$	2
$\pi/4$	1.41
$\pi/2$	1
$3\pi/4$	1.41
$5\pi/6$	2



Since the cosine function is 0 at odd multiples of $\frac{\pi}{2}$, the secant function has vertical asymptotes at odd multiples of $\frac{\pi}{2}$.

x	f(x)
$-\pi/3$	2
$-\pi/4$	1.41
0	1
$\pi/4$	1.41
$\pi/3$	2
$2\pi/3$	-2
$3\pi/4$	-1.41
π	-1
$5\pi/4$	-1.41
$4\pi/3$	-2



All material has been taken from *Precalculus,* by M. Sullivan, 10th Edition