Section 7.2 – The Inverse Trigonometric Functions (Continued)

In Examples 6 and 7 in Sec 7.1, we solved compound problems using Method 1, finding the angle θ. It is not necessary to be able to find the angle θ in order to solve this type of compound problem. Method 2 is to use side lengths to solve, rather than finding angle θ.

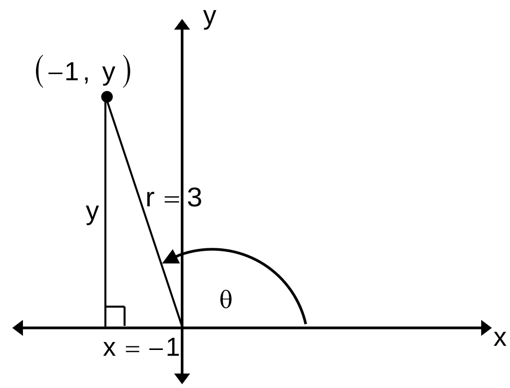
Example 8: Find the exact value of $\sin\left(\cos^{-1}\left(\frac{-1}{3}\right)\right)$.

Method 2

Let $\theta = \cos^{-1}\left(\frac{-1}{3}\right),\ 0 \le \theta \le \pi$.

$\Rightarrow \cos\theta = \frac{-1}{3},\ 0 \le \theta \le \pi$

Since $\cos\theta < 0$, it follows that $\frac{\pi}{2} < \theta \le \pi$, $\Rightarrow \theta$ in Quad II



$\cos\theta = \frac{-1}{3}$

$= \frac{x}{r}$

$\Rightarrow$ Assume $x = -1, r = 3$

Then $x^2 + y^2 = r^2$

$(-1)^2 + y^2 = 3^2$

$1 + y^2 = 9$

$y^2 = 8$

Quad II, y positive: $y = \sqrt{8}$

$= 2\sqrt{2}$

So, $\sin\left(\cos^{-1}\left(\frac{-1}{3}\right)\right) = \sin\theta$

$= \frac{y}{r}$

$= \frac{2\sqrt{2}}{3}$

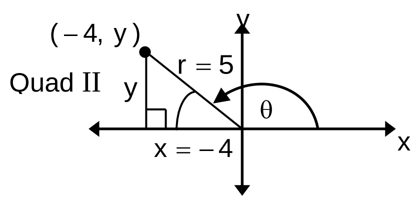
Example 9: Find the exact value of $\sin\left[\cos^{-1}\left(\frac{-4}{5}\right)\right]$.

Method 2

Let $\theta = \cos^{-1}\left(\frac{-4}{5}\right),\ 0 \le \theta \le \pi$.

$\Rightarrow \cos\theta = \frac{-4}{5},\ 0 \le \theta \le \pi$.

$\cos\theta$ negative $\Rightarrow \theta$ in Quad II



$\cos\theta = \frac{-4}{5}$

$= \frac{x}{r}$ Assume $x = -4, r = 5$

$x^2 + y^2 = r^2$

$(-4)^2 + y^2 = 5^2$

$16 + y^2 = 25$

$y^2 = 9$

$y = \sqrt{9}$ Quad II, y positive

$y = 3$

Thus, $\sin\left[\cos^{-1}\left(\frac{-4}{5}\right)\right] = \sin\theta$

$= \frac{y}{r}$

$= \frac{3}{5}$

Section 7.2 – The Inverse Trigonometric Functions (Continued)

The Remaining Inverse Trigonometric Functions

The inverse cotangent of x:

$y = \cot^{-1} x$ means $x = \cot y$

where $-\infty < x < \infty$ and $0 < y < \pi$.

The inverse cosecant of x:

$y = \csc^{-1} x$ means $x = \csc y$

where $|x| \ge 1$ and $\frac{-\pi}{2} \le y \le \frac{\pi}{2}$, $y \ne 0$.

The inverse secant of x:

$y = \sec^{-1} x$ means $x = \sec y$

where $|x| \ge 1$ and $0 \le y \le \pi$, $y \ne \frac{\pi}{2}$.

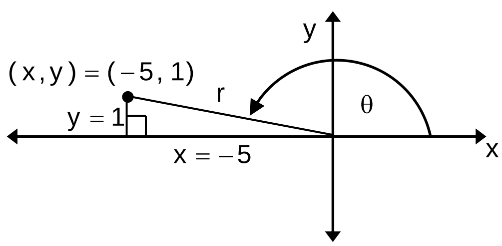
Your calculator does not have keys for evaluating these inverse trigonometric functions. To evaluate them, convert them to an inverse trigonometric function whose range is the same as the one to be evaluated. Except where they are undefined, $y = \cot^{-1} x$ and $y = \sec^{-1} x$ have the same range as $y = \cos^{-1} x$, while $y = \csc^{-1} x$ has the same range as $y = \sin^{-1} x$.

Example 10: Use a calculator to approximate $\cot^{-1}(-5)$ in radians rounded to two decimal places.

Let $\theta = \cot^{-1}(-5)$, $0 < \theta < \pi$.

$\Rightarrow \cot\theta = -5$, $0 < \theta < \pi$. $\cot\theta$ is negative $\Rightarrow \theta$ in quadrant II $\left(\frac{\pi}{2} < \theta < \pi\right)$

$= \frac{x}{y}$ $\Rightarrow$ Assume $x = -5$, $y = 1$



$x^2 + y^2 = r^2$

$(-5)^2 + 1^2 = r^2$

$25 + 1 = r^2$

$r^2 = 26$

$r = \sqrt{26}$

Thus, $\cos\theta = \frac{x}{r}$, $\frac{\pi}{2} < \theta < \pi$ (Use cosine, not sine, since the range of cosine inverse is essentially the same as that of cotangent inverse)

$= \frac{-5}{\sqrt{26}}$

$= \frac{-5\sqrt{26}}{26}$

So, $\theta = \cos^{-1}\left(\frac{-5\sqrt{26}}{26}\right)$.

Thus, $\cot^{-1}(-5) = \theta$

$= \cos^{-1}\left(\frac{-5\sqrt{26}}{26}\right)$

≈ 2.944

≈ 2.94 $\left(\text{and } \frac{\pi}{2} < 2.94 < \pi\right)$

All material has been taken from *Precalculus,* by M. Sullivan, 10th Edition