Section 7.3 – Trigonometric Equations – Day 1

In this section you will look at **trigonometric equations**, which are equations involving trigonometric functions that are satisfied only by some values, or possibly no values, of the variable. The values that satisfy the equation are called **solutions** of the equation.

Unless the domain of the variable is restricted, you need to find all the solutions of a trigonometric equation. To find all the solutions, first find solutions over an interval whose length equals the period of the function and then add multiples of that period to the solutions found.

Example 1: Solve the equation $\cos\theta = \frac{\sqrt{3}}{2}$.

The period of the cosine function is 2π.

In the interval $[0,\ 2\pi)$, there are two angles for which

$\cos\theta = \frac{\sqrt{3}}{2}$: $\theta = \frac{\pi}{6}$ and $\theta = \frac{11\pi}{6}$

The equation has an infinite number of solutions due to the periodicity of the cosine function.

Because the cosine has period 2π, all the solutions of $\cos\theta = \frac{\sqrt{3}}{2}$ may be given by

$\theta = \frac{\pi}{6} + 2k\pi$ and $\theta = \frac{11\pi}{6} + 2k\pi$, where k is any integer.

To check your solution, you may graph $y_1 = \cos x$ and $y_2 = \frac{\sqrt{3}}{2}$, in radian mode. The solutions are where the graphs intersect.

Example 2: Solve the equation $\sin(2\theta) = 1,\ 0 \le \theta < 2\pi$.

The period of the sine function is 2π.

In the interval $[0,\ 2\pi)$, the sine function has the value 1 only at an angle of $\frac{\pi}{2}$.

So, $2\theta = \frac{\pi}{2} + 2k\pi$, k any integer.

$\Rightarrow \theta = \frac{\pi}{4} + k\pi$, k any integer.

But the domain is restricted to $0 \le \theta < 2\pi$, so it follows that $\theta = \frac{\pi}{4}$ and $\theta = \frac{5\pi}{4}$ are the only solutions.

To check your solution, you may graph $y_1 = \sin(2x)$ and $y_2 = 1$, in radian mode. The solutions are where the graphs intersect.

In using a calculator to solve trigonometric equations, remember that the calculator supplies an angle only within the restrictions of the definition of the inverse trigonometric function. To find the remaining solutions, you must identify other quadrants, if any, in which the angle may be located.

Example 3: Use a calculator to solve $\cos\theta = 0.4$ for $0 \le \theta < 2\pi$.

$\Rightarrow \theta = \cos^{-1}(0.4)$

Your calculator gives $\theta \approx 1.159279$, an angle in the first quadrant. But, cosine is also positive in quadrant IV.

The angle $2\pi - \theta = 2\pi - 1.159279$

≈ 5.123906 is the angle in quadrant IV where $\cos\theta = 0.4$.

Thus, the solution to $\cos\theta = 0.4$ for $0 \le \theta < 2\pi$ is $\theta \approx 1.16$ and $\theta \approx 5.12$ radians.

Section 7.3 – Trigonometric Equations – Day 1 (continued)

Many trigonometric equations can be solved by factoring or by applying the quadratic formula. When a trigonometric equation contains more than one trigonometric function, identities may be used to create an equivalent equation that contains only one trigonometric function. If you square both sides of an equation, remember to check your solutions because extraneous solutions may be introduced.

Example 4: Solve the equation $3\cos\theta + 3 = 2\sin^2\theta,\ \ 0 \le \theta < 2\pi$.

$3\cos\theta + 3 = 2\sin^2\theta$

$3\cos\theta + 3 = 2(1-\cos^2\theta)$ Use the Pythagorean Identity

$3\cos\theta + 3 = 2 - 2\cos^2\theta$

$2\cos^2\theta + 3\cos\theta + 1 = 0$ This is a quadratic equation in $\cos\theta$

$(2\cos\theta + 1)(\cos\theta + 1) = 0$ Factor

$\Rightarrow\ 2\cos\theta + 1 = 0$ or $\cos\theta + 1 = 0$ By the Zero Product Property

$2\cos\theta = -1$ | $\cos\theta = -1$

$\cos\theta = \dfrac{-1}{2}$ | $\Rightarrow \theta = \pi + 2k\pi$, k any integer

$\Rightarrow \theta = \dfrac{2\pi}{3} + 2k\pi$, k any integer

and

$\theta = \dfrac{4\pi}{3} + 2k\pi$, k any integer

General Solutions

On the interval $[0,\ 2\pi)$, the solutions are $\theta = \dfrac{2\pi}{3},\ \theta = \pi,$ and $\theta = \dfrac{4\pi}{3}$.

Example 5:

Solve the equation $\cos^2\theta + \sin\theta = 4$.

$\cos^2\theta + \sin\theta = 4$

$\left(1 - \sin^2\theta\right) + \sin\theta = 4$ Pythagorean Identity

$-\sin^2\theta + \sin\theta - 3 = 0$

$\sin^2\theta - \sin\theta + 3 = 0$

This is a quadratic equation in $\sin\theta$.

Discriminant $b^2 - 4ac = (-1)^2 - 4(1)(3)$

$= 1 - 12$

$= -11$

< 0

$\Rightarrow$ There is no real solution.

Example 6:

Solve the equation: $\sin(2\theta) = \dfrac{1}{2},\ \ 0 \le \theta < 2\pi$.

$\sin(2\theta) = \dfrac{1}{2}$

Sine is positive in quadrants I and II, so

$2\theta = \dfrac{\pi}{6} + 2k\pi$ and $2\theta = \dfrac{5\pi}{6} + 2k\pi$, k any integer

$\Rightarrow \theta = \dfrac{\pi}{12} + k\pi$ and $\theta = \dfrac{5\pi}{12} + k\pi$, k any integer

(General Solutions)

On the interval $[0,\ 2\pi)$, the solutions are

$\theta = \dfrac{\pi}{12},\ \theta = \dfrac{5\pi}{12},\ \theta = \dfrac{13\pi}{12},$ and $\theta = \dfrac{17\pi}{12}$.

All material has been taken from *Precalculus,* by M. Sullivan, 10th Edition