Section 7.5 – Trigonometric Equations – Day 2

Use Sum and Difference Formulas to establish Identities

Another use of the Sum and Difference Formulas is to establish other identities. Two important identities are given next.

$$\cos\left(\frac{\pi}{2}-\theta\right)=\sin\theta \quad \text{and} \quad \sin\left(\frac{\pi}{2}-\theta\right)=\cos\theta$$

Example 6: To prove $\cos\left(\frac{\pi}{2}-\theta\right)=\sin\theta,$ use the formula for $\cos(\alpha-\beta)$ with $\alpha=\frac{\pi}{2}$ and $\beta=\theta.$

$$\begin{aligned}\cos\left(\frac{\pi}{2}-\theta\right)&=\cos\frac{\pi}{2}\cos\theta+\sin\frac{\pi}{2}\sin\theta\\&=\frac{x}{r}\cos\theta+\frac{y}{r}\sin\theta \qquad \frac{\pi}{2}:(0,1)\\&=\frac{0}{1}\cos\theta+\frac{1}{1}\sin\theta\\&=(0)\cos\theta+(1)\sin\theta\\&=0+\sin\theta\\&=\sin\theta\end{aligned}$$

Example 7: To prove $\sin\left(\frac{\pi}{2}-\theta\right)=\cos\theta,$ use the identity just established

$$\begin{aligned}\sin\left(\frac{\pi}{2}-\theta\right)&=\cos\left[\frac{\pi}{2}-\left(\frac{\pi}{2}-\theta\right)\right]\\&=\cos\theta\end{aligned}$$

Example 8: Establish the identity: $\frac{\cos(\alpha-\beta)}{\sin\alpha\sin\beta}=\cot\alpha\cot\beta+1$

$$\begin{aligned}\frac{\cos(\alpha-\beta)}{\sin\alpha\sin\beta}&=\frac{\cos\alpha\cos\beta+\sin\alpha\sin\beta}{\sin\alpha\sin\beta}\\&=\frac{\cos\alpha\cos\beta}{\sin\alpha\sin\beta}+\frac{\sin\alpha\sin\beta}{\sin\alpha\sin\beta}\\&=\frac{\cos\alpha}{\sin\alpha}\left(\frac{\cos\beta}{\sin\beta}\right)+1\\&=\cot\alpha\cot\beta+1\end{aligned}$$

$$\therefore \frac{\cos(\alpha-\beta)}{\sin\alpha\sin\beta}=\cot\alpha\cot\beta+1$$

Example 9: Establish the identity:

$$\tan(\theta+\pi)=\tan\theta$$

$$\begin{aligned}\tan(\theta+\pi)&=\frac{\tan\theta+\tan\pi}{1-\tan\theta\tan\pi}\\&=\frac{\tan\theta+\frac{y}{x}}{1-\tan\theta\left(\frac{y}{x}\right)}\\ \text{Now, } \pi:(1,0) \qquad &=\frac{\tan\theta+\frac{0}{1}}{1-\tan\theta\left(\frac{0}{1}\right)}\\&=\frac{\tan\theta+0}{1-\tan\theta(0)}\\&=\frac{\tan\theta}{1-0}\\&=\frac{\tan\theta}{1}\\&=\tan\theta\end{aligned}$$

$$\therefore \tan(\theta+\pi)=\tan\theta$$

This confirms the tangent function is periodic with period π.

All material has been taken from *Precalculus,* by M. Sullivan, 10th Edition